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TITLE: UNDERSTANDING AND APPLYING HARRY MARKOWITZ'S PORFOLIO THEORY

Supervisor: Professor GABRIELE CARDULLO

Student: CHAKO YOTCHEU ELIETTE DORIA

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ABSTRACT

Modern Portfolio Theory (MPT), introduced by Harry Markowitz in 1952, revolutionized finance by providing a structured framework for crafting investment portfolios that balance risk and return. This study surveys MPT's evolution, from its inception to contemporary applications, covering seminal contributions, empirical investigations, and extensions. It emphasizes MPT's foundational concepts, including the efficient frontier and the link between risk and reward. The Capital Asset Pricing Model (CAPM) is explored for its connection between asset attributes and market dynamics.

Empirical studies assess MPT-based portfolios' real-world effectiveness, especially during market turbulence. Critiques are examined, including challenges to assumptions about investor rationality and market efficiency, with behavioral finance highlighting psychological biases' impact on decision-making. Technological advancements in computation and data analytics enhance portfolio precision and risk assessment.

MPT's application in global and emerging markets, including considerations of currency risk and ESG integration, is discussed. Ultimately, this study underscores MPT's enduring relevance in investment analysis, adapting to the evolving financial landscape through theory, research, and contemporary applications.

La Modern Portfolio Theory (MPT), introdotta da Harry Markowitz nel 1952, ha rivoluzionato il settore finanziario fornendo un quadro strutturato per la creazione di portafogli di investimento che bilanciano il rischio e il rendimento. Questo studio analizza l'evoluzione della MPT, dalla sua nascita alle applicazioni contemporanee, coprendo contributi seminali, indagini empiriche ed estensioni. Si mettono in evidenza i concetti fondamentali della MPT, tra cui la frontiera efficiente e il collegamento tra rischio e rendimento. Viene esplorato il Modello di Valutazione degli Asset Finanziari (CAPM) per la sua connessione tra attributi degli asset e dinamiche di mercato.

Studi empirici valutano l'efficacia dei portafogli basati sulla MPT nel mondo reale, specialmente durante le turbolenze di mercato. Si esaminano le critiche, inclusi i dubbi sulle assunzioni legate alla razionalità degli investitori e all'efficienza di mercato, con la finanza comportamentale che evidenzia l'impatto dei bias psicologici sul processo decisionale. Gli avanzamenti tecnologici nella computazione e nell'analisi dei dati migliorano la precisione nella costruzione del portafoglio e nella valutazione del rischio.

Viene discusso l'utilizzo della MPT nei mercati globali ed emergenti, inclusa la considerazione del rischio valutario e dell'integrazione di criteri ambientali, sociali e di governance (ESG). In definitiva, questo studio sottolinea la duratura rilevanza della MPT nell'analisi degli investimenti, adattandosi al mutevole panorama finanziario attraverso teoria, ricerca empirica e applicazioni contemporanee.

CHAPTER Ⅰ: MODERN PORTFOLIO THEORY

1.1 Background

Modern Portfolio Theory (MPT), pioneered by Harry Markowitz in 1952, profoundly transformed the landscape of financial economics by introducing a structured framework for crafting investment portfolios that optimize the delicate balance between risk and return. This comprehensive study presents an exhaustive survey of the evolutionary journey and practical applications of MPT since its inception, spanning seminal contributions, empirical investigations, extensions beyond conventional models, and the assimilation of cutting-edge computational tools and behavioral insights.

Central to this analysis is the meticulous explication of MPT's foundational tenets, notably the concept of the efficient frontier and the critical interplay between risk and reward. This exploration extends further to encompass the Capital Asset Pricing Model (CAPM), a framework that intricately connects the risk and return attributes of individual assets with broader market-wide dynamics. Moreover, the assessment delves into empirical inquiries that gauge the real-world efficacy of portfolios designed according to MPT principles, particularly their resilience amidst market turbulence and financial crises.

Within this review lies an incisive examination of the critiques lodged against MPT, rigorously probing assumptions tied to investor rationality, market efficiency, and the dependability of risk and correlation estimations. Notably, the integration of behavioral finance takes center stage, illuminating the profound impact of psychological biases on the decision-making processes behind portfolio management. Noteworthy advancements in computational methodologies and data analytics also assume prominence, showcasing how these technological strides facilitate the precision of portfolio construction and risk evaluation.

As financial landscapes undergo an era of heightened globalization, this investigation navigates the application of MPT principles in the context of international and emerging markets. It further contemplates intricate considerations such as currency risk and geopolitical nuances. Moreover, the emergent trend of Environmental, Social, and Governance (ESG) integration garners scrutiny, spotlighting the adaptability of MPT frameworks to accommodate ethical imperatives in investment choices.

In summation, this exhaustive survey firmly establishes Modern Portfolio Theory as an enduring cornerstone of investment analysis, perpetually evolving alongside the fluid contours of the financial realm. The synthesis of its theoretical bedrock, empirical inquiries, and contemporary applications offers an all-encompassing comprehension of MPT's abiding significance within the modern financial milieu.

1.2 Literature review

The financial market is a meeting place for the supply and demand of financial assets where investors engage through taking long and short positions in various financial instruments. In the modern portfolio theory, investor preferences are defined in terms of return and risk.

A portfolio's return is a combination of the returns of the assets it contains, weighted by their proportions in the portfolio. Risk is a function of the correlation among the assets that constitute it.

Therefore, it is crucial to diversify one's portfolio to avoid significant fluctuations in security prices due to multiple financial shocks and an increase in geopolitical and macroeconomic uncertainties. These fluctuations are also linked to the overall health of the sector in which one invests. It is well known that sectors like technology and telecommunications experience substantial fluctuations. Diversification follows the famous portfolio management adage, "Don't put all your eggs in one basket."

According to the modern portfolio theory developed by Markowitz (1952), agents aim to combine a set of assets with maximum return at a given level of risk or equivalently, minimal risk for a given level of return. This is the efficient portfolio.

Harry Markowitz's work led to the theorization of optimal diversification of a stock portfolio. These works operate on assumptions:

- Investment occurs over a single period (e.g., 6 months or 1 year).
- There are no transaction costs.
- Investor preferences consider only two criteria: risk and expected return.
- Markets are efficient.
- Investors have risk aversion.

Regarding the choice of risk measure, several alternatives are available. For instance, in Markowitz's (1952) portfolio management model, the portfolio's risk is determined by its variance. Thus, all deviations, negative or positive, from the expected return are considered. The main outcome of this model stipulates that at the optimum, the portfolio held by the investor should be perfectly diversified.

Dominating for over half a century, Markowitz's (1952) model remains one of the most widely used by practitioners. It relies on the strong assumption that agents possess a quadratic utility function and uses the standard deviation (or variance) of security returns to measure a portfolio's risk.

The Markowitz model does not define a single optimal portfolio but generates an efficient frontier comprising all optimal portfolios. It is up to the investor to select their optimal portfolio based on their level of risk aversion.

A literature review on Modern Portfolio Theory (MPT) encompasses an extensive exploration of research, theories, and advancements related to this influential framework. Since its introduction by Harry Markowitz in 1952, MPT has revolutionized investment analysis by providing a systematic approach to constructing portfolios that balance risk and return. Here's a comprehensive overview of the key themes and developments in the MPT literature:

Foundational Contributions: The literature typically commences with a recognition of Harry Markowitz's seminal work, where he introduced the concept of diversification and the efficient frontier. This initial research laid the groundwork for MPT by demonstrating how investors could optimize returns for a given level of risk through diversification.

Efficient Frontier and Risk-Return Tradeoff: A central theme of the literature revolves around the efficient frontier, which illustrates all portfolios that offer the highest expected return for a specified level of risk. Researchers delve into how investors can allocate their investments along this frontier to achieve their desired risk-return profile.

Capital Asset Pricing Model (CAPM): William Sharpe's introduction of the Capital Asset Pricing Model in 1964 extended MPT by linking individual asset risk and return to market-wide risk and return. This model provides a foundation for understanding asset pricing and portfolio diversification in relation to market dynamics.

Extensions and Critiques: Researchers often explore extensions of MPT to include factors beyond traditional risk and return, such as liquidity, inflation, and behavioral biases. The literature also critically examines MPT's assumptions, including the efficient market hypothesis and investors' rational behavior.

Empirical Testing and Applications: A substantial portion of the literature involves empirical studies that test the effectiveness of MPT in real-world scenarios. Researchers assess the historical performance of portfolios constructed using MPT principles, often analyzing how these portfolios perform during market downturns and periods of heightened volatility.

Advanced Techniques and Computational Tools: With advancements in computing technology, researchers have integrated advanced techniques into MPT, including mathematical optimization, simulation methods, and machine learning algorithms. The literature explores how these tools enhance portfolio construction and risk management.

Behavioral Finance and MPT: Behavioral finance have gained traction in the MPT literature, investigating how psychological biases and irrational behavior impact portfolio decisions. Researchers explore ways to incorporate behavioral insights into the MPT framework.

Portfolio Management and Asset Allocation Strategies: The practical application of MPT principles in portfolio management and asset allocation is a significant focus. The literature discusses various strategies for constructing diversified portfolios, including passive indexing, active management, and the incorporation of alternative investments.

Global Portfolio Management and Emerging Markets: With the increasing interconnectedness of financial markets, the literature explores applying MPT principles to global portfolio management. Considerations such as currency risk, international diversification, and geopolitical factors are examined in this context.

Environmental, Social, and Governance (ESG) Integration: The integration of ESG factors into MPT frameworks is a growing trend. The literature examines how MPT can be adapted to incorporate ethical and sustainability considerations into portfolio construction and risk assessment.

Therefore, a literature review on Modern Portfolio Theory delves into its foundational concepts, empirical studies, contemporary applications, and evolving adaptations. It underscores MPT's enduring significance in investment theory and practice, while acknowledging the ongoing evolution of the theory within the dynamic landscape of modern finance.

1.3 Statistical measures

A portfolio, in the context of finance, is meticulously crafted by grouping various assets together, with utmost consideration for the investor's risk aversion, utility function, budgetary limitations, and the available information. Effective portfolio management necessitates the ability to foresee market trends, enabling portfolio managers to meticulously assess securities based on diverse criteria following a comprehensive analysis.

These selection criteria predominantly revolve around three core factors: expected return, risk evaluation typically quantified through variance, and the distribution's return dispersion. The portfolio manager's role entails meticulously ranking securities based on these dimensions, thus ensuring the optimal alignment of the portfolio with the investor's risk-return preferences.

After this scrutiny, the portfolio manager proceeds to an equally pivotal phase: asset allocation. This phase marks the initial and foremost step in the portfolio management process. Here, the portfolio manager strategically allocates assets across the investment spectrum, bearing in mind the investor's individual risk tolerance, financial goals, and market insights. This allocation process inherently shapes the portfolio's risk and potential returns, thereby influencing the overall investment trajectory.

Concurrently, the aspect of ongoing vigilance assumes paramount significance. Monitoring the portfolio is an essential endeavor that aims to fulfill the investor's stipulated return targets while adhering to their constraints and accounting for the risk they are willing to shoulder. The investment landscape is ever evolving, and as such, maintaining a diligent watch over the portfolio's performance becomes pivotal in ensuring its alignment with the investor's financial aspirations.

Hence, the portfolio management process is an intricate interplay of careful analysis, strategic allocation, and continuous vigilance. It hinges upon the capacity to harmonize the investor's preferences and constraints with market dynamics, ultimately seeking to achieve optimal returns while prudently managing risk exposure.

1) Expected returns.

In the Markowitz model of Modern Portfolio Theory (MPT), expected returns are a crucial element in the calculation of portfolio optimization. Expected returns represent the anticipated average returns of different assets in the portfolio. Notations used in the Markowitz model include:

$\boldsymbol{E(Ri)}$**:** Expected returns of asset i

Wi: Weight of asset i in the portfolio, indicating the proportion of the total investment allocated to that asset. The weights collectively determine the composition of the portfolio.

$\boldsymbol{R_P}$: Expected return of the entire portfolio. It is calculated as the weighted sum of the expected returns of the individual assets in the portfolio.

$$\boldsymbol{R_P = \Sigma_i w_i \cdot E(R_i)}$$

$cov(R_i, R_j)$ Covariance between the returns of asset i and asset j. This measures the degree to which the returns of the two assets move together. A positive covariance indicates they tend to move in the same direction, while a negative covariance suggests they move in opposite directions.

$\boldsymbol{\sigma i}$: Standard deviation of asset $i's$ returns. It quantifies the variability or dispersion of returns around the expected return. It is a measure of the asset's risk or volatility.

$Corr(R_i, R_j)$: Correlation coefficient between the returns of asset i and asset j. It's a normalized measure of the linear relationship between the returns of the two assets, ranging from -1 (perfect negative correlation) to 1 (perfect positive correlation).

In the context of portfolio construction, the investor's objective is to maximize the portfolio's expected return for a given level of risk or minimize the portfolio's risk for a given level of expected return. This involves selecting the appropriate weights for each asset in the portfolio. The expected return of the portfolio is a linear combination of the expected returns of the individual assets, weighted according to their proportions in the portfolio.

By incorporating these notations and concepts, investors can construct portfolios that optimize the trade-off between risk and expected return, resulting in portfolios positioned along the efficient frontier of possible portfolios.

2) Variance:

Within the framework of the Markowitz model in Modern Portfolio Theory, variance assumes a pivotal role in the assessment of portfolio risk and potential returns. Variance acts as a metric that gauges the extent of dispersion or spread within a set of values, most notably the returns of distinct assets within a set of values, most notably the returns of distinct assets within a portfolio. In the context of the Markowitz model, variance's notational representations are as follows:

$\boldsymbol{\sigma_i^2}$**:** This signifies the variance of the returns associated with asset i. It encapsulates the average squared deviation between the asset's actual returns and their mean (expected return). A higher variance value typically indicates a broader dispersal of returns, a characteristic often associated with elevated risk levels.

$\boldsymbol{\sigma_{ij}}$ Referring to the covariance between the returns of asset $i \text{ and asset } j$, this metric quantifies the degree to which the returns of the two assets move together. Covariance is an essential element in the computation of portfolio variance.

$\boldsymbol{\sigma_p^2}$: Representing the variance of the entire portfolio, it considers the weighted sum of individual asset variances and covariances between assets. It offers insight into how the overall returns of the portfolio are distributed.

In the context of the Markowitz model, the portfolio's risk is evaluated based on the portfolio's variance, which considers the variances of the individual assets and their pairwise covariances. A key principle of portfolio optimization is to find the combination of asset weights that minimizes the portfolio's variance while achieving a desired level of expected return. This optimization process helps investors find portfolios that provide the best possible trade-off between risk and expected return along the efficient frontier.

3) Covariance.

In the realm of the Markowitz model within Modern Portfolio Theory, covariance assumes a great role in dissecting the interplay between the returns of distinct assets. This dynamic measurement underpins the construction of diversified portfolios and the meticulous assessment of associated risks. Covariance articulates the way in which the returns of two assets align – whether in concert (positive covariance) or contrast (negative covariance). The standard notation for expressing covariance within the Markowitz model is:

$cov(R_i, R_j)$: This notation captures the covariance between the returns of asset i *and asset* j. It quantifies the degree to which the movements in returns of these two assets coincide. Positive covariance signifies their propensity to move in a similar direction, while negative covariance denotes an inverse relationship in their returns.

Covariance's significance lies in its capacity to unravel the interactions among assets within a portfolio. Elevated covariance suggests synchronized return patterns, which could contribute to heightened portfolio volatility. Conversely, low or negative covariance fosters diversification benefits by indicating that the assets' returns do not align, thereby helping to mitigate overall portfolio risk.

1.4 The concept of utility and risk

In the Markowitz model of Modern Portfolio Theory, the concepts of utility and risk are intricately interlinked and form the foundation for constructing efficient investment portfolios.

Utility: Utility embodies the notion of investor satisfaction and preference when it comes to different combinations of risk and return in their investment choices. It quantifies how much value an investor places on potential gains and how averse they are to potential losses. Utility functions mathematically capture these preferences and risk attitudes.

Risk: Risk signifies the inherent uncertainty or variability of returns associated with various investment assets. The Markowitz model aims to manage and minimize this risk while simultaneously maximizing potential returns. Risk is often measured using statistical tools such as variance and standard deviation, which provide insights into the potential fluctuations in asset returns. Two important categories of risk exist:

Specific Risk (Unsystematic Risk): Specific risk, also known as unsystematic risk, refers to risk factors that are unique to a particular company or asset. These risks are inherent to the specific characteristics of the investment and are not related to broader market movements. Specific risk can be mitigated through diversification – by investing in a variety of assets, an investor can reduce the impact of specific risk on the overall portfolio.

Examples of specific risk include company-specific events like management changes, product recalls, legal disputes, and supply chain disruptions. Since these risks are specific to individual companies, they can be minimized by holding a diversified portfolio that includes assets from different industries and sectors.

Systematic Risk (Market Risk): Systematic risk, also known as market risk, is the risk that is inherent to the entire market or economy. It's beyond the control of individual investors or companies and affects all investments to some degree. Systematic risk is related to broad economic factors, geopolitical events, interest rate changes, and other macroeconomic variables that impact the overall market.

Unlike specific risk, systematic risk cannot be eliminated through diversification. It is a fundamental part of investing and is not unique to any particular investment. Systematic risk influences the performance of all assets in the market and cannot be mitigated by holding a diversified portfolio alone.

Examples of systematic risk include economic recessions, inflation, changes in government policies, and global events that affect financial markets as a whole.
Both types of risk play a role in portfolio management, and understanding their distinctions helps investors make informed decisions about risk management and asset allocation.

The model recognizes the essential trade-off between risk and return, whereby investors generally anticipate higher returns for assuming higher levels of risk. This relationship, however, is nonlinear; as risk escalates, the incremental potential return diminishes. Consequently, the model endeavors to achieve an optimal equilibrium between risk and return, ensuring that portfolios are well-balanced and aligned with investor preferences.

Diversification emerges as a key strategy within the Markowitz model to manage risk effectively. By holding a diversified mix of assets with different risk profiles, investors can mitigate the potential impact of poor performance from individual assets on the overall portfolio.

We can distinguish different types of risk preference of each individual /investor. Risk preferences refer to an individual's inclination or attitude towards taking on various levels of risk in their investment decisions. Different people have different risk preferences, and understanding these preferences is crucial for constructing portfolios that align with their comfort zones. Here are the Common types of risk preferences:

Risk-Averse**:** Risk-averse individuals have a lower tolerance for risk and prioritize the preservation of capital over the pursuit of high returns. They are more concerned about potential losses and are willing to accept lower expected returns in exchange for a higher degree of safety. Risk-averse investors often prefer investments with lower volatility, such as bonds and stable dividend-paying stocks.

Risk-Neutral**:** Risk-neutral individuals are indifferent to risk and are solely focused on maximizing expected returns. They make decisions purely based on the potential for higher gains, without considering the level of risk involved. This type of risk preference is more common in theoretical financial models than in real-world behavior.

Risk-Seeking (or Risk-Loving): Risk-seeking individuals actively seek out riskier investments in the pursuit of higher returns. They are willing to embrace high volatility and the potential for significant losses if there's a chance for substantial gains. This risk preference is relatively rare and often associated with speculative behavior.

So, understanding your own risk preference is essential when making investment decisions. It helps you select investments and construct portfolios that not only align with your comfort level but also aim to achieve your financial objectives. It's worth noting that risk preferences are personal and can be influenced by factors such as age, financial situation, investment knowledge, and psychological traits.

Here's how utility and risk are connected within the context of the Markowitz model:

Risk and Return Trade-Off: The Markowitz model recognizes that there is a trade-off between risk and return. Investors generally expect higher returns for taking on higher levels of risk. However, this relationship is not linear; as risk increases, the additional potential return diminishes. The model aims to find the optimal balance between risk and return to achieve the best possible portfolio outcome.

Risk Measurement: Risk is often quantified using statistical measures, with variance and standard deviation being commonly used. Variance measures the average squared deviation of asset returns from their mean return, while standard deviation is the square root of variance. A higher variance or standard deviation indicates higher risk and greater potential fluctuations in returns and vice versa.

Portfolio Diversification: The Markowitz model emphasizes the importance of diversification in managing risk. By holding a mix of assets with different risk profiles, investors can reduce the overall risk of the portfolio. Diversification helps mitigate the impact of poor performance from one asset on the entire portfolio.

Covariance and Correlation: The relationship between different assets' returns is captured by covariance and correlation. Positive covariance indicates that two assets tend to move in the same direction, while negative covariance suggests opposite movements. Correlation further standardizes this relationship, giving a value between -1 (perfect negative correlation) and 1 (perfect positive correlation).

Efficient Frontier: The efficient frontier is a key concept in the Markowitz model that represents the set of portfolios that provide the highest return for a given level of risk or the lowest risk for a given level of return. Portfolios on the efficient frontier are optimal in the sense that no other portfolio offers a better risk-return trade-off.

Optimal Portfolio: An optimal portfolio is one that lies on the efficient frontier and is tailored to an investor's risk tolerance and return expectations. The Markowitz model helps investors identify the optimal mix of assets that minimizes risk while maximizing potential returns.

In essence, the concepts of utility and risk intertwine to shape portfolio decisions. Utility functions quantify an investor's preferences and risk aversion, enabling them to make informed choices that align with their individual risk-return trade-offs. The model's emphasis on minimizing risk while maximizing potential returns aids investors in constructing portfolios that are well-positioned to achieve their financial objectives.

1.5 Budget constraints

In the context of the Markowitz model of Modern Portfolio Theory, budget constraints refer to the limitations on the total amount of capital available for investment. Budget constraints play a significant role in portfolio optimization, as they determine the allocation of funds across different assets while adhering to the available investment resources. Here's how budget constraints are incorporated into the Markowitz model:

Total Available Capital and Investment allocation: The total amount of capital available for investment is a critical input in the Markowitz model. It represents the maximum amount of money an investor has to allocate among various assets to construct a portfolio. The objective of portfolio optimization is to allocate the available capital across a set of investment assets in a way that maximizes returns or minimizes risk. The allocation is determined by the weights assigned to each asset in the portfolio.

Budget Equation: The budget equation is a fundamental equation in the Markowitz model that ensures the sum of the allocated funds across all assets does not exceed the total available capital. This equation ensures that the investor's capital is fully utilized, and no assets are over- or under allocated.

Constraints and Efficient Frontier: The budget constraint, along with risk and return expectations, influences the shape and position of the efficient frontier. The efficient frontier represents the optimal set of portfolios that offer the best risk-return trade-off for a given level of risk or return. The budget constraint restricts the combinations of assets that can be chosen to construct the portfolio.

Risk and Return Trade-Off: Within the budget constraint, an investor aims to allocate funds to assets in a way that balances the trade-off between risk and return. The investor's preferences, risk tolerance, and return expectations guide the selection of asset allocations that maximize utility while adhering to the available capital.

Optimal Portfolio: The optimal portfolio is the one that lies on the efficient frontier and maximizes returns or minimizes risk while satisfying the budget constraint. This portfolio achieves the best possible risk-return trade-off within the investor's resource limitations.

To wrap up, budget constraints are integral to the MPT model's portfolio optimization process. They ensure that the allocation of funds across different assets is realistic and aligned with the investor's available capital. The model seeks to construct portfolios that balance risk and return while adhering to these budgetary limitations.

CHAPTER Ⅱ: MARKOWITZ MODEL

2.1 Assumptions

The Markowitz model, a cornerstone of Modern Portfolio Theory, is based on a set of underlying assumptions that serve as the building blocks of its framework. These assumptions simplify the real-world complexities of financial markets and enable the mathematical formulation of portfolio optimization. Here's a comprehensive overview of the key assumptions in the Markowitz model:

Investment Objectives: Investors make decisions primarily based on two factors: the expected return and risk associated with their investments. These attributes are pivotal in constructing portfolios that strike an optimal balance between potential rewards and possible losses.

Market Efficiency: The Markowitz model operates under the assumption of efficient markets, where asset prices promptly reflect all available information. This concept implies that investors cannot consistently outperform the market by exploiting historical data or publicly available information.

Rational Investor Behavior: Investors are considered rational and risk averse. This implies that investors seek higher expected returns while avoiding excessive risk. Their decisions are guided by maximizing their utility based on the trade-off between risk and return.

Inclusion of Risk-Free Asset: The model incorporates a risk-free asset, typically represented by government bonds. This risk-free asset offers a known rate of return and can be combined with risky assets to construct efficient portfolios along the Capital Market Line.

Neglect of Taxes and Costs: Taxes and transaction costs are not taken into consideration within the model. While this simplification eases calculations, real-world investments are influenced by these additional factors.

Single-Period Focus: The model operates within a single investment period and does not delve into multi-period considerations or sequential decision-making. This assumption streamlines the analysis by focusing solely on the current period's parameters.

Uniform Expectations: It is assumed that all investors possess uniform expectations regarding the returns and risks of various assets. This uniform interpretation of historical data and pertinent information allows for standardized calculations.

Stable Expected Returns and Covariances: The model assumes that expected returns and covariances (or correlations) between assets remain constant throughout the investment period. These values can change due to shifts in economic conditions and other variables.

Normal Distribution of Returns: The model presupposes that asset returns follow a normal distribution, which simplifies the mathematical computations. However, actual market returns often exhibit more intricate distribution patterns.

These assumptions collectively establish the foundation upon which the Markowitz model is built. While these simplifications help in conducting quantitative analyses and constructing efficient portfolios, it's important to recognize that these assumptions might not capture the full complexity of real-world market dynamics. Despite these limitations, the Markowitz model remains an influential tool in understanding how risk and return interact within investment portfolios.

2.2 Determination of the optimal portfolio

The theory revolves around the concept of diversification and finding the optimal portfolio that balances risk and returns. Here’s how you can determine the optimal portfolio in the Markowitz model:

Data Collection: Gather historical data on the returns of various assets you’re considering for your portfolio. This data should cover a reasonable time period to capture different market conditions. This could include stocks, bonds, commodities and other investments. Here’s is a more detailed breakdown of the data collection process:

-Selecting Assets, decide on the types of assets you want to consider for your portfolio. These could include stocks, bonds, mutual funds, exchange-traded funds (ETFs), commodities, and other investment instruments.

-Time Period, Determine the time period for which you want to collect data. This time period should be long enough to capture different market conditions and economic cycles.

-Data Sources, identify reliable sources from which you can obtain historical price or return data for the selected assets. Common sources include financial databases, brokerage platforms, financial news websites, and economic research organizations.

-Frequency of Data, decide on the frequency of data you need. Daily, weekly, or monthly data are commonly used. The choice of frequency may depend on the assets you're analyzing and the level of granularity you require.

-Data Point, for each asset, collect the relevant data points. These data points typically include the closing prices or total returns (including dividends or interest) for each chosen time interval.

-Calculate Returns, Use the collected price or total return data to calculate the periodic returns for each asset. The return for a specific period can be calculated using the formula: Return = $(Ending\ Price - Beginning\ price)/(Beginning\ Price)$

Where:

Return is the return for the chosen period.

Ending price is the price of the asset at the end of the period.

Beginning price is the price of the asset at the beginning of the period.

-Calculate the Expected returns, Covariance and Correlation

-Data Preprocessing, Clean the collected data to remove any outliers or erroneous values that could distort the analysis. Inconsistent or missing data points should also be addressed through imputation or interpolation.

-Update Data Regularly, to keep your portfolio optimization up-to-date, regularly update the historical data. New data points should be incorporated to reflect changing market conditions and economic dynamics.

By following these steps, you'll be able to collect accurate and relevant data that forms the foundation for the portfolio optimization process in the Markowitz model. Remember that the quality of the data you collect greatly influences the accuracy of your portfolio optimization results.

Construct the Efficient Frontier: The efficient frontier is a graph that shows the optimal portfolios that provide the highest expected return for a given level of risk. To construct the efficient frontier, you need to simulate different portfolios by allocating different weights to the available assets. For each portfolio, calculate the expected return and standard deviation (a measure of risk). Plot these points on a graph to form the efficient frontier.

Select the Optimal Portfolio: The optimal portfolio is the point on the efficient frontier that corresponds to your risk tolerance and investment goals. If you're a risk-averse investor, you might select a portfolio with lower risk and slightly lower expected return. If you're more risk-tolerant, you might opt for a portfolio with higher expected return and higher risk.

Tangency Portfolio or Capital Market Line: The point where the efficient frontier touches a line drawn from the risk-free rate is known as the tangency portfolio. This point represents the optimal combination of the risk-free asset and the risky portfolio on the efficient frontier. The slope of this line is called the Sharpe ratio, which measures the excess return earned per unit of risk.

Allocating between Risky and Risk-Free Assets: Depending on your risk preference, you can choose an allocation between the tangency portfolio and the risk-free asset. This allocation will determine the final composition of your portfolio.

Monitor and Rebalance: Portfolios need to be monitored and periodically rebalanced to maintain their desired asset allocation. Market conditions and the performance of individual assets can cause the portfolio to deviate from its intended allocation over time.

Remember that the Markowitz model provides a valuable framework for understanding the trade-off between risk and return in investment decisions.

2.3 Efficient Frontier

Within the realm of portfolio selection, the Markowitz Model holds substantial significance, albeit primarily within theoretical boundaries. As a quantitative approach, it equips investors with the tools to adeptly manage their resources by deftly navigating the intricate equilibrium between risk and reward. This framework acts as a compass for streamlining financial resources effectively, achieved by scrutinizing the inherent risk and potential returns inherent in an investment portfolio. Consequently, the model emerges as a catalyst for judicious decision-making, empowering investors to meticulously tailor asset allocation and individual investments for optimal outcomes.

The Efficient Frontier, in this context, serves as a pivotal concept. It stands as a guiding light for investors, facilitating the identification of optimal portfolio allocations based on their distinct risk tolerance and return objectives. Graphically, the Efficient Frontier takes shape as a spectrum of potential combinations of assets, each offering the most favorable return potential for varying levels of risk.

This visual construct materializes by plotting a diverse array of portfolios on a graph. On this canvas, the x-axis delineates the portfolio's standard deviation, a measure of its risk profile, while the y-axis embodies the portfolio's expected return. The resulting scatter plot reveals an ensemble of data points, each signifying a unique portfolio composition arising from distinct asset allocations.

In essence, the Efficient Frontier embodies the quintessence of portfolio optimization. It empowers investors to strategically navigate the trade-off between risk and reward, ultimately fostering a harmonious synergy between these pivotal facets of investment strategy.

All portfolios lying on or above the Efficient Frontier are considered efficient because they provide the maximum possible return for the level of risk, or the minimum possible risk for the level of return.

Portfolios below the Efficient Frontier are considered suboptimal because they offer lower expected returns for the same level of risk or higher risk for the same level of return compared to portfolios on the Efficient Frontier.

The point on the Efficient Frontier that touches a line drawn from the risk-free rate is known as the tangency portfolio. This point represents the optimal combination of the risky portfolio (Efficient Frontier) and the risk-free asset, creating the highest possible Sharpe ratio.

The slope of the line from the risk-free rate to the tangency portfolio is the Sharpe ratio. It measures the excess return per unit of risk and helps investors assess the desirability of different portfolios.

The Efficient Frontier underscores the importance of diversification. By combining assets with different risk and return characteristics, investors can achieve a portfolio that lies on the Efficient Frontier, thus optimizing their risk-return profile.

Depending on an investor's risk tolerance, they can choose a point on the Efficient Frontier that aligns with their preferences. Risk-averse investors may opt for portfolios with lower risk and slightly lower returns, while risk-seeking investors might select portfolios with higher returns and higher risk.

The Efficient Frontier is not static; it shifts with changes in asset prices and correlations. As a result, investors need to periodically rebalance their portfolios to maintain their desired allocation and keep their portfolios efficient.

The Efficient Frontier assumes that asset returns follow a normal distribution and that correlations remain constant. These assumptions may not always hold true in real-world markets.

The Efficient Frontier provides a visual framework for investors to make informed decisions about portfolio construction, considering their unique risk preferences and return objectives. It's a tool that allows for a systematic approach to achieving a balanced and optimal portfolio composition within the Markowitz Model.

An example of the efficient frontier in the Markowitz model can be seen in the following figure:

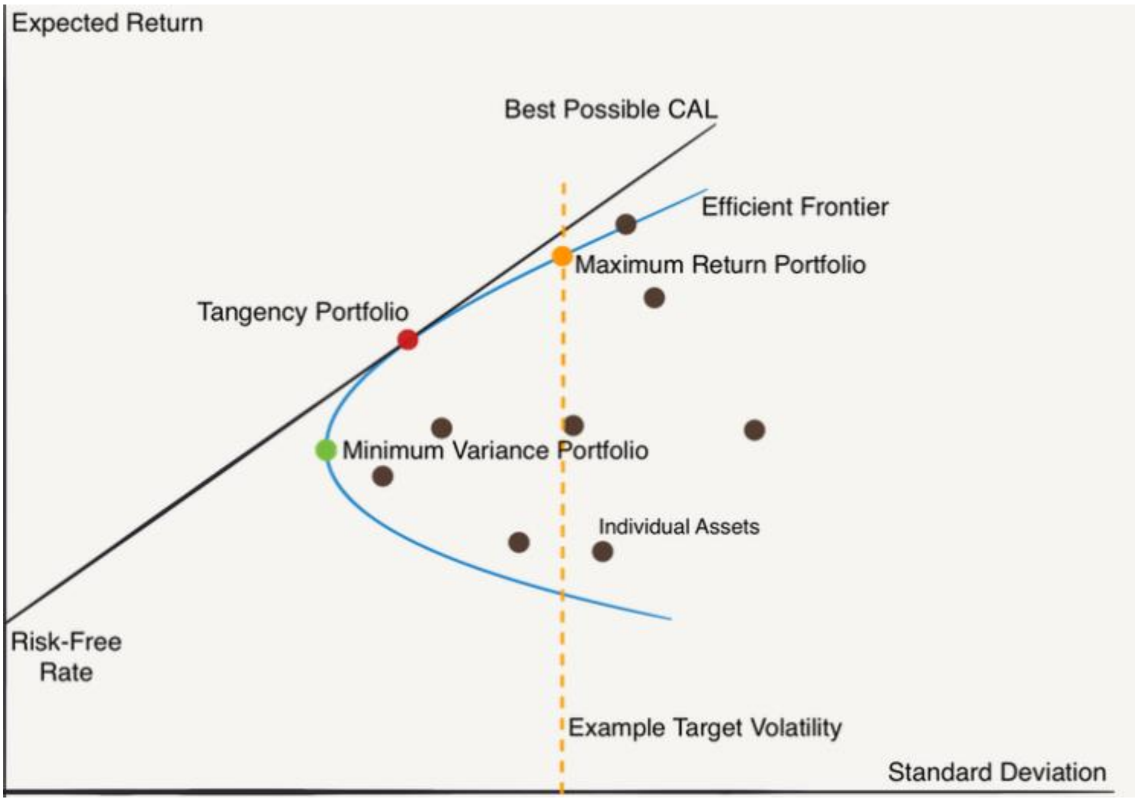


Fig: 2.3.1 Efficient Frontier

Within the confines of the efficient frontier (excluding the immediate boundary), lie individual assets or various combinations thereof that fall short of optimization.

The tangency portfolio, indicated by the red point in the depicted image, embodies the acclaimed optimal portfolio that achieves the utmost Sharpe ratio attainable. Progressing either leftward or rightward along the frontier from this juncture would yield a diminished Sharpe ratio, denoting a decrease in the excess return-to-risk ratio.

The juncture at which the hyperbola transitions from convex curvature to concave contour designates the location of the minimum variance portfolio, marked by the green point in the provided image.

At a specific volatility level, there additionally exists a portfolio known as the maximum return portfolio, depicted by the orange point in the illustration. True to its name, this portfolio optimizes returns while maintaining the designated volatility level.

Now, let's introduce the linear Capital Market Line (CML). The intersection of the CML with the Y-axis designates the return corresponding to an investor's risk-free asset, such as government securities. This line tangentially aligns with the efficient frontier precisely at the point of the Maximum Sharpe portfolio. The CML (tangency) line consequently portrays a portfolio comprising diverse blends of a risk-free asset and a tangency portfolio, also referred to as the maximum Sharpe portfolio or occasionally termed the "optimal portfolio."

When delving into the mathematical aspects, an efficient frontier can be precisely calculated – analytically – solely under the simplest circumstance where there's only one constraint, such as the sum of asset weights equating to one. However, as you introduce additional constraints on asset weights, the computation of the frontier necessitates the application of an optimization process. As a result of these constraints, the frontier might deviate from its initial hyperbolic form, taking on varying curves in alignment with the new constraints. While specific scenarios might yield explicit solutions, this is not generally the case.

A straightforward approach to computing the efficient frontier while adhering to constraints is through the utilization of Markowitz's Critical Line Algorithm (CLA). Unlike certain quadratic optimization methods, this technique remains effective even when the quantity of assets far exceeds the number of observations. The core premise of CLA revolves around several uncomplicated steps. Initiated by selecting the asset with the highest return, positioned on the upper right segment of the Efficient Frontier, the process involves tracing the Frontier to the left. This entails identifying the subsequent optimal asset for inclusion or gradual removal, one step at a time.

2.4 Diversification

The terms "diversification" and "Diversification Effect" encompass the interplay between correlations and portfolio risk. Diversification, a pivotal principle within Markowitz's portfolio selection theory and Modern Portfolio Theory (MPT), revolves around reducing risk. It involves allocating investments across diverse financial instruments, industries, and investment categories, resonating with the adage of not putting all one's eggs in a single basket. In simpler terms, the idea is that by spreading investments, the risk of losing everything is considerably diminished. This concept is exemplified by the metaphor of placing eggs in multiple baskets; if one basket is dropped, not all eggs are lost.

Diversification is realized through investments in various stocks, asset classes (such as bonds and real estate), and commodities like gold or oil. The objective is to optimize returns and minimize risk by investing in assets that would react differently to the same events. For instance, adverse news related to the European debt crisis might lead to a decline in the stock market, but simultaneously, certain commodities like gold may gain value due to a flight to safety. Effective portfolio diversification encompasses not only different stocks within and outside the same industry but also various asset classes like bonds and commodities.

The Diversification Effect arises from imperfect correlations between assets. This effect is instrumental in risk reduction since it allows for risk mitigation without sacrificing returns. Savvy investors who are risk-averse naturally gravitate toward diversification strategies. This is particularly important given that the correlation between assets can influence portfolios in meaningful ways, enabling risk management without sacrificing overall investment gains. Let's explore how diversification operates within the framework of the Markowitz Model:

Risk Reduction: The core premise of diversification in the Markowitz Model is the recognition that investors generally seek to maximize returns while minimizing risk. However, risk and return are inherently linked, meaning that achieving higher returns often involves taking on more risk. Diversification exploits the fact that different assets may have dissimilar patterns of returns, correlations, and sensitivities to market changes.

By combining assets with varying risk profiles, their individual fluctuations tend to offset each other. As a result, the overall portfolio's volatility (risk) can be lower than the average volatility of its constituent assets.

Shared volatility and Interdependence: The Markowitz Model accounts for the covariance (or correlation) between asset returns. Assets with low or negative correlations offer the most effective diversification benefits because they move independently of each other. This reduced co-movement helps mitigate the impact of negative events affecting one asset on the entire portfolio.

Optimal portfolio frontier: The Efficient Frontier is the graphical representation of the optimal portfolios that provide the highest expected return for various levels of risk. The Markowitz Model demonstrates that by combining assets with different risk-return profiles, investors can achieve portfolios that lie on or above the Efficient Frontier, thereby maximizing returns for a given level of risk.

Maximum Sharpe ratio portfolio: The tangency portfolio, found at the point where the Capital Market Line touches the Efficient Frontier, embodies the highest Sharpe ratio and optimal risk-return trade-off. It reflects a well-diversified blend of risky assets and a risk-free asset, demonstrating how diversification contributes to superior portfolio performance.

2.4.1 Benefits and Limitations

Some benefits of this Strategy are that Diversification helps to:

- Lower overall portfolio risk by reducing the impact of individual asset volatilities.

- Mitigate the impact of unexpected events or poor performance of individual assets

- Provide a more stable and predictable investment journey by smoothing out extreme fluctuations.

- Enhance risk-adjusted returns by finding the best balance between risk and return.

- Preserve Capital meaning diversification helps protect capital, even if one investment suffers a significant loss, the impact on the entire portfolio is less severe, preserving the investor's capital.

- Accumulate wealth in long term, diversification is a key strategy for long-term wealth accumulation. It helps investors stay invested through various market cycles, allowing their investments to compound over time.

- Customize, diversification can be tailored to meet an investor's specific risk tolerance, investment horizon, and financial goals. This allows for a customized approach to portfolio management.

While diversification offers numerous advantages and benefits, it also has its limitations and potential drawbacks that investors should be aware of:

-It does not eliminate all types of risk. Systemic risks, market downturns, and unexpected correlations can still affect diversified portfolios.

-Additionally, over-diversification can dilute potential return. Diversification may limit the potential for exceptionally high returns. When a portfolio is highly diversified, it is less likely to include significant holdings in individual assets that experience exceptional gains. Therefore, the portfolio's overall return may be more moderate.

- Transaction Costs and complexity: Maintaining a well-diversified portfolio can lead to higher transaction costs, especially for frequent trading or rebalancing. These costs can eat into overall returns, particularly for smaller portfolios. Managing a diversified portfolio can be complex, especially for individual investors. Keeping track of multiple assets, their performance, and the need for rebalancing can be time-consuming and challenging. It's possible to over-diversify, where adding more assets to a portfolio doesn't provide significant risk reduction benefits but increases complexity and may lead to mediocre returns. Finding the right balance is crucial.

-*Homogenous markets*: Diversifying within the same asset class or sector may not provide true diversification. For example, owning multiple technology stocks may not protect against sector-specific risks.

-*Changing Market Dynamics*: Markets are dynamic, and what worked for diversification in the past may not work in the future. Market conditions and asset correlations can change over time.

-*Market correlation:* Diversification may not always be effective during times of extreme market stress or systemic events. During such periods, asset correlations tend to rise, and most assets may decline together, limiting the risk-reduction benefits of diversification.

The Markowitz Model highlights that diversification is a cornerstone of effective portfolio management, enabling investors to navigate the intricate interplay between risk and return. By constructing portfolios that harness the benefits of diversification, investors can achieve more stable and optimized investment outcomes. In summary, diversification is a valuable risk management strategy, but it's essential to recognize its limitations. Investors should carefully consider their investment objectives, risk tolerance, and the specific assets in their portfolio to strike the right balance between risk reduction and potential returns. Additionally, diversification should be viewed as part of a broader risk management and investment strategy.

2.5 Strengths and Limitations of the model

The Markowitz Model, also known as Modern Portfolio Theory (MPT), has several strengths that have made it a fundamental framework for portfolio management and investment analysis:

The Markowitz model explicitly addresses the trade-off between risk and return. It helps investors understand that, in general higher returns are associated with higher risk, and vice versa. This fundamental insight guides investment decisions.

The model provides a rigorous and quantitative framework for portfolio optimization. It uses statistical analysis and mathematical optimization to determine the optimal asset allocation.

The Markowitz model emphasizes the importance of diversification in reducing portfolio risk. It quantifies how combining assets with different risk profiles can lead to a more efficient and less risky portfolio. This refers to diversification benefits of the model.

The model introduces the concept of efficient frontier, which represents a set of portfolios that offer the highest expected return for a given level of risk or the lowest risk for a given level of return. This graphical representation makes it easy to visualize the risk-return trade-off.

MPT introduces the Sharpe ratio and other risk adjusted performance metrics. These ratios allow investors to evaluate the performance of a portfolio relative to its risk. This is particularly important for comparing different investment options.

It provides a systematic approach to portfolio construction. It encourages investors to select assets and allocate capital based on data and analysis rather than intuition or emotion.

The model is flexible and can accommodate various types of assets, including stocks, real estate, and more. It can be applied to different investment horizons and objectives. Investors can also customize their portfolios according to their risk tolerance and return objectives. This allows for tailored solutions that align with individual goals.

It encourages a long-term perspective on investing. By considering risk over extended periods, it aligns with the idea of building wealth steadily over time. It also incorporates risk management as a core component of portfolio construction. It helps investors identify and manage risk factors in their portfolios.

The principals of the Markowitz model are widely applicable and have relevance beyond traditional investments. They can be applied to various decision-making contexts involving risk and uncertainty. It laid the foundation for modern finance and influenced the development pf other financial theories and models such as the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT).

Despite these strengths, it's important to note that the Markowitz Model also has limitations and assumptions, including the assumptions of normal distribution of asset returns and constant correlations, which may not always hold in real world markets. Therefore, while it provides valuable insights, it should be used in conjunction with other analytical tools and considerations for comprehensive portfolio management.

The Markowitz Model, while a foundational framework for portfolio optimization, has several limitations and assumptions that can affect its practical applicability:

Normal Distribution Assumption: The model assumes that asset returns follow a normal distribution, which implies that extreme events are rare. In real world, financial markets can experience extreme events or "fat-tailed" distributions, which the model does not account for adequately.

Constant Correlations: The Markowitz Model assumes that asset correlations remain constant over time. In practice, correlations can vary, especially during times of economic stress or systemic events.

Single-Period Model: The model is primarily a single-period model, meaning it focuses on optimizing portfolios for a specific time frame. It does not address dynamic changes in investment strategies or the effects of portfolio decisions over multiple periods.

Sensitivity to Input Data: Portfolio optimization results in the Markowitz Model can be sensitive to input data, such as expected returns, standard deviations, and correlations. Small changes in these inputs can lead to significantly different portfolio allocations.

No Consideration of Transaction Costs: The model does not account for transaction costs associated with buying and selling assets. Frequent trading can erode returns, and the model may suggest unrealistic levels of portfolio turnover.

Lack of Short Selling Constraints: The Markowitz Model does not inherently impose constraints on short selling (selling assets that an investor does not own), which may not align with some investors' preferences or restrictions.

Difficulty in Estimating Parameters: Estimating parameters such as expected returns and correlations can be challenging and subject to estimation errors, particularly for assets with limited historical data.

Ignores Behavioral Factors: The model assumes that investors are solely motivated by rational, utility-maximizing behavior and do not consider behavioral biases or psychological factors that can influence decision-making.

Risk Tolerance Uncertainty: The model requires investors to specify their risk tolerance as a precise input, which can be challenging as risk tolerance can vary over time and in response to changing market conditions.

Does Not Address Non-Traditional Assets: The Markowitz Model was developed primarily for traditional asset classes like stocks and bonds. It may not be well-suited for alternative investments, such as cryptocurrencies or private equity.

Out-of-Sample Performance: Portfolio optimization based on historical data may not perform as well in out-of-sample (future) periods, as market conditions and relationships can change.

Overemphasis on Short-Term Volatility: The model places significant emphasis on short-term volatility (standard deviation) as a measure of risk, which may not align with the long-term investment horizon of some investors.

Despite these limitations, the Markowitz Model remains a valuable tool for understanding the principles of portfolio diversification and risk-return trade-offs. Many of its shortcomings have been addressed and extended by subsequent portfolio optimization models that attempt to address some of these limitations, such as the consideration of transaction costs or the modeling of non-normally distributed returns. Additionally, Practitioners often use the Markowitz Model as a starting point and then adjust better to reflect real-world conditions and investor preferences.

CHAPTER Ⅲ: METHODOLOGY AND DATA ANALYSIS

3.1 Methodological approach

Research on the Markowitz model is highly relevant and important because it empowers individuals, institutions, and policymakers to make better-informed decisions in the world of finance. It provides valuable insights into portfolio management, risk management and investment strategy, contributing to more efficient and responsible financial practices.

The data I used to conduct my analysis came from different financial and investment websites such as: Just ETF, Yahoo and Google finance, Bloomberg, Interactive Broker and from the official website of the Bank of Italy.

Here's an overview of how I collected and analyzed the data used:

-I defined the research objective which was to get the return of the past 10 years of different classes of assets such as bonds, class of Stock and Cryptocurrencies. In particular, I analyzed the return of the Italian BTP (Buoni del Tesoro Poliennale), of Bitcoin and ETFs.

-I proceed by selecting my data sources which included historical data of my chosen asset classes from different online sources.

-Then I used the data collection method known as the Secondary Data, which refers to data that is publicly available.

-Finally, I gathered the data collected and analyzed them using excel.

The process of data analysis was done in different steps:

-The so-called Exploratory Data Analysis (EDA) i.e., it's an initial step to understand the data. It involves creating visualizations for example charts, and graphs and summary statistics such as mean, standard deviation, to identify patterns, outliers and potentials relationships within the data.

It's important to approach data collection and analysis with rigor, transparency, and ethical considerations. Additionally, the specific methods and tools used for data collection and analysis will vary depending on the research discipline and objectives.

THE CHOSEN CLASSES OF ASSETS

A class of assets refers to a grouping or category of financial assets that share similar characteristics or are subject to comparable accounting and reporting standards. These asset classes are instrumental in the fields of finance and accounting for categorizing and organizing various asset types, facilitating analysis, reporting, and investment management. Asset classes aid investors and organizations in making well-informed decisions regarding resource allocation and investment strategy. Commonly recognized asset classes encompass:

- Equities (Stocks): This class includes ownership of shares in publicly traded companies. Equity investments represent ownership in a company and may provide returns through dividends and capital appreciation.

- Fixed-Income Securities (Bonds): This class consists of debt instruments issued by governments, corporations, or other entities. Bondholders receive periodic interest payments and the return of the principal amount at maturity.

- Real Estate: Real estate assets include physical properties such as residential, commercial, and industrial properties. Real estate investments can generate rental income and appreciate in value.

-Cash and Cash Equivalents: This class includes liquid assets like cash, bank deposits, and short-term investments with maturities of three months or less. They are highly liquid and low risk.

- Commodities: Commodities are physical goods like oil, gold, and agricultural products. Investors can buy and sell commodity futures contracts or invest in commodity-related stocks and funds.

- Alternative Investments: This category covers a wide range of non-traditional investments, such as hedge funds, private equity, venture capital, and cryptocurrencies. These assets often have unique risk-return profiles.

- Foreign Currencies: Investors can hold foreign currencies or engage in currency trading in the foreign exchange (Forex) market.

-Derivatives: This class includes financial contracts, such as options and futures, that derive their value from an underlying asset. Derivatives are often used for hedging and speculation.

Asset classes serve as a framework for diversifying a portfolio to manage risk and achieve specific financial goals. Diversification across asset classes can help spread risk and potentially enhance returns.

The most common class of assets used in investment varies depending on individual preferences, investment goals, and economic conditions. However, historically, the two most common and widely recognized asset classes for investment are:

Equities, often referred to as stocks, hold a prominent position in the investment landscape. People choose to invest in stocks for various compelling reasons, and these motivations are interlinked in their pursuit of financial growth and security.

First and foremost, equities represent ownership shares in publicly traded companies. As one of the most prevalent investment options for both individuals and institutions, stocks offer the potential for long-term capital appreciation. This potential for growth over time is a driving force behind investors' decisions to enter the stock market. By investing in stocks, individuals aim to see the value of their holdings increase over time, providing an opportunity to sell them at a higher price than their initial purchase.

Furthermore, stocks offer another avenue for generating income: dividend payments. Many publicly traded companies distribute a portion of their profits to shareholders in the form of dividends. This dividend income serves as an attractive prospect for investors seeking regular, periodic payments from their investments, thus adding an income stream to their portfolio.

In the broader context of portfolio management, stocks are recognized as a fundamental element. Diversification is a key strategy, and stocks play a pivotal role in achieving it. By investing in a variety of stocks across different industries and sectors, investors can effectively spread risk. This diversification strategy mitigates the impact of poor performance in any one stock or sector, contributing to a more balanced and resilient investment portfolio.

Moreover, investing in stocks isn't just about financial gain; it's about ownership in companies. When you buy stocks, you acquire a share of a company. This aspect appeals to some investors who relish the idea of being partial owners and having a say in company matters, often through voting rights attached to certain shares.

Stocks also boast a distinct advantage in terms of liquidity. They are highly liquid assets, facilitating easy buying and selling on stock exchanges. This liquidity provides investors with the flexibility to access their investments when needed, promoting adaptability in their financial strategies.

Additionally, the allure of long-term growth is a driving factor behind stock investments. Investors with a horizon that extends far into the future often choose stocks as a core component of their wealth-building strategy. Over extended periods, stocks have historically delivered higher average returns compared to many other asset classes, making them a favored choice for those with a patient investment approach.

For investors seeking professional guidance and management of their portfolios, there are investment vehicles like mutual funds and exchange-traded funds (ETFs). These funds are expertly managed by professionals and hold diversified portfolios of stocks. This approach enables investors to benefit from the seasoned decision-making of investment experts without the need to individually select and manage stocks.

In summary, investing in stocks offers a multitude of interconnected benefits, from capital appreciation and dividend income to portfolio diversification, ownership in companies, liquidity, and the potential for long-term growth. These factors collectively make stocks a versatile and attractive option for those looking to build and secure their financial futures.

However, investors interested in investing in stocks should conduct thorough research, assess their risk tolerance, and consider their long-term financial goals when making investment decisions. Diversification and a well-thought-out investment strategy are often key components of a successful stock investment approach.

MY PORTFOLIO COMPOSITION:

1.ETFS

Exchange-Traded Funds (ETFs) are investment funds that are traded on stock exchanges, much like individual stocks. They are designed to track the performance of a specific index, commodity, bond, or a basket of assets, such as stocks or bonds. ETFs offer investors a way to gain exposure to a diversified portfolio of assets or a particular segment of the market without having to buy each individual security separately. Some reasons why I decided to choose ETFs instead of an individual stock are as follows:

-Diversification, ETFs typically hold a diversified portfolio of assets. For example, an S&P 500 ETF will aim to replicate the performance of the S&P 500 index by holding a proportionate share of the 500 constituent stocks. This diversification helps spread risk.

-Liquidity and Transparency, ETFs are traded on stock exchanges throughout the trading day, just like individual stocks. This means investors can buy or sell ETF shares at market prices during trading hours. The high liquidity of ETFs makes them easy to buy and sell. ETFs disclose their holdings on a daily basis, allowing investors to see exactly which assets are held within the fund. This transparency provides clarity and helps investors make informed decisions.

-Low Costs and Flexibility, ETFs are often known for their low expense ratios compared to many mutual funds. Lower costs can lead to higher returns for investors over time. They cover a wide range of asset classes, including equities (stocks), fixed income (bonds), commodities, real estate, and more. This versatility allows investors to tailor their portfolios to their specific investment goals and risk tolerances.

-Intraday Trading, ETFs can be bought and sold throughout the trading day at market prices, providing investors with the ability to make real-time investment decisions and employ trading strategies like limit orders(it sets a maximum price that you are willing to pay or a minimum price that you are willing to accept on a sale) and stop orders(it's triggered / activated when an asset reaches a certain price and filled at the next available price and are not visible to the market).

-Professional Management, while many ETFs are passively managed and aim to replicate the performance of an underlying index, there are also actively managed ETFs that have professional portfolio managers making investment decisions.

-Niche Exposure, ETFs can provide access to niche markets and sectors that may be difficult or expensive to access through individual securities. This includes sectors like emerging markets, specific industries, or unique investment strategies.

Overall, ETFs have become popular investment vehicles due to their versatility, cost-effectiveness, and ease of use. Investors can use ETFs as building blocks for constructing diversified portfolios, managing risk, and achieving specific investment objectives.

2. ITALIAN GOVERNMENT BONDS (Buoni del Tesoro Poliennali, BTP)

Bonds are debt instruments issued by governments, corporations, or other entities. Investors in bonds receive periodic interest payments (coupon payments) and the return of the principal amount at maturity. Bonds are known for their relative stability compared to stocks and are often used to generate regular income and reduce portfolio risk.

The decision to include Italian BTP in my portfolio was supported for various reasons, and bonds are an essential component of many investment portfolios. Here are some common motivations for investing in bonds:

-Income Generation and Preservation of Capital, Italian BTP provide regular interest payments, known as coupon payments, to investors. These payments can serve as a stable and predictable source of income, making bonds attractive to investors seeking regular cash flow. They are generally considered less volatile than stocks. Investors often turn to bonds to preserve their capital and protect their investments from the market's ups and downs. Bonds are seen as a safer investment option in comparison to equities.

-Diversification and Risk mitigation, they offer diversification benefits to portfolios that are heavily weighted toward stocks or other asset classes. By including bonds in a portfolio, investors can reduce overall risk and potentially enhance long-term returns. Government and high-quality corporate bonds are often viewed as low-risk investments. Investors, especially those with a low risk tolerance, may invest in bonds to reduce their exposure to the inherent volatility of the stock market.

-Steady Returns and Safety, Italian BTP have historically provided relatively stable and predictable returns. Investors who prioritize capital preservation and a steady income stream may find bonds to be an attractive investment choice. Those issued by governments or highly rated corporations are considered safer investments because they are less likely to default on their interest and principal payments. This safety aspect can be especially appealing to conservative investors.

-Tax Benefits, some types of bonds, such as municipal bonds in the United States, offer tax advantages. Interest income from these bonds may be exempt from federal and sometimes state income taxes, making them attractive to investors in higher tax brackets.

-Diversification Across Asset Classes, they are considered a separate asset class from stocks and can help diversify a portfolio. This diversification can spread risk and reduce the impact of poor performance in any one asset class.

-Fixed Maturity Date and Safeguarding Against Inflation, Btp have a fixed maturity date when the principal is repaid. This can make them suitable for investors with specific financial goals, such as funding a child's education or a major purchase, at a future date. While inflation can erode the purchasing power of money, some bonds, such as Treasury Inflation-Protected Securities (TIPS), are indexed to inflation. Investing in such bonds can help protect against the eroding effects of inflation.

In summary, people invest in bonds for a range of reasons, including income generation, capital preservation, risk mitigation, diversification, steady returns, safety, retirement planning, liquidity, tax benefits, and protection against inflation. However, it's crucial to note that Italian BTPs are not without risks. Like all investments, they come with their own set of potential downsides, including interest rate risk, inflation risk, and credit risk though this is generally low for government bonds in developed countries like Italy. But these risks are lower compared to risk encounter in stocks and other assets. The choice to invest in bonds is often influenced by individual financial goals, risk tolerance, and the desire for a balanced and well-diversified investment portfolio.

The choice between these two asset classes, as well as other asset classes like real estate, cash and cash equivalents, commodities, and alternative investments, depends on various factors, including an investor's risk tolerance, investment horizon, income needs, and overall financial goals. Diversification across multiple asset classes is a common strategy to spread risk and potentially enhance returns in an investment portfolio. The specific allocation to each asset class should be based on an investor's unique circumstances and investment strategy.

3.CRYPTOCURRENCY (BITCOIN)

Bitcoin is the third class of asset I included in my portfolio and the reasons are quite diversified. Since Markowitz's core principle is diversification, which involves spreading your investment across different asset classes to reduce risk. Bitcoin is often seen as a non-correlated or low correlated asset compared to traditional stocks and bonds. If Bitcoin exhibits low or negative correlation with the existing portfolio assets, it can be an effective tool for risk reduction through diversification. This is particularly valuable during times of market turbulence when asset correlations tend to increase. By including Bitcoin in my portfolio, it's possible to achieve greater diversification and potentially reduce the overall risk.

His theory highlights the tradeoff between risk and return. Bitcoin, due to its volatility, is considered a high-risk asset. However, it has also shown the potential for high returns. By including Bitcoin, you can potentially increase the overall return potential of your portfolio, but this comes with an acceptance of higher risk.

Markowitz's theory is built on the idea of long-term investing and holding diversified portfolios. If you believe in the long-term potential of Bitcoin and its ability to diversify your portfolio, you can consider including it as a long-term strategic allocation.

Potential for High Returns and Inflation Hedge, Bitcoin has demonstrated the potential for significant price appreciation over relatively short periods. While it is known for its volatility, this volatility can also present opportunities for substantial gains. Some investors are attracted to Bitcoin for its potential to deliver high returns. They are often referred to as "digital gold" or "digital store of value" because, like gold, it is viewed by some as a store of value that can protect against inflation and Investors looking to preserve their wealth and protect their assets from economic uncertainty may find Bitcoin appealing. Its fixed supply (limited to 21 million coins) and decentralized nature make it attractive in times of economic uncertainty and potential currency devaluation. Bitcoin operates on a decentralized blockchain network, which means it is not controlled by any single entity, government, or central bank. Some investors value this decentralization as a way to protect against government interference or manipulation. When you own Bitcoin, you have direct ownership and control over your assets. You can hold and manage your Bitcoin independently, without the need for intermediaries like banks or brokers.

While these advantages make a compelling case for including Bitcoin in a portfolio, it's essential to acknowledge the associated risks and challenges. Bitcoin's price volatility, regulatory uncertainty, lack of intrinsic value, and potential for loss are factors that should be carefully considered.

TIME FRAME

The choice of a 10-year period (2012-2022) for analyzing the historical return of my chosen assets was because a 10-year period provides a relatively long-term perspective on an asset's performance. It allows investors to evaluate how the asset has fared through different market cycles, economic conditions, and events. Also, because, A 10-year period offers a sufficient sample size of historical data points to make meaningful statistical analyses. It helps smooth out short-term fluctuations and noise in asset prices, providing a more reliable picture of performance. Economic and market cycles typically unfold over several years. A 10-year period encompasses multiple cycles, which can be valuable for assessing how an asset performs during both bull (upward) and bear (downward) markets. It's important to note that while a 10-year analysis provides valuable insights into an asset's historical performance, it should not be the sole basis for making investment decisions. Investors should consider a range of factors, including the asset's fundamentals, current market conditions, and their own financial goals and risk tolerance. Additionally, historical performance does not guarantee future results, and past trends may not necessarily continue in

the future. Therefore, comprehensive due diligence and analysis are essential when evaluating investments.

3.2 Data presentation

*ETFs

I collected the data of these ETFs thanks to a website known as “Just ETFs”.

The Lyxor FTSE MIB UCITS ETF - Dist replicates the FTSE MIB index. The FTSE MIB index replicates the 40 largest Italian companies. The Lyxor FTSE MIB UCITS ETF - Dist is the largest ETF that replicates the FTSE MIB index. The FTSE MIB index serves as Italy's primary stock market benchmark, encompassing the 40 most prominent and highly liquid Italian blue-chip corporations. ETF replicates the performance of the underlying index with full physical replication (buying all components of the index). The ETF's dividends are distributed to investors (Annually). The FTSE MIB (Milano Indice di Borsa) Index is a prominent stock market benchmark that monitors the performance of Italy's top 40 largest and most liquid companies listed on the Borsa Italiana. This index is characterized by a free-floating structure and is weighted by market capitalization. The initial reference point for the FTSE MIB Index was established at 10,644 points, mirroring the closing level of the MIB 30 Index on October 31, 2003.

In the context of stock market indices like the FTSE MIB, "free-floating" refers to the methodology used to determine the index's composition. A free-floating index typically includes only the shares of a company that are available for trading in the open market. This excludes shares that are closely held by company insiders, government entities, or other entities that do not actively trade their shares. It focuses on the portion of a company's shares that are in the hands of public investors and are available for buying and selling on the stock exchange. These publicly traded shares are often referred to as "float" or "free float."

The rationale behind using free-floating shares in an index is to provide a more accurate representation of the market's sentiment and the tradable portion of a company's equity. It reflects the shares that are actively bought and sold by investors, which can have a more significant impact on the index's performance and the overall market.

Therefore, when an index like the FTSE MIB is characterized as "free-floating," it means that it includes only the publicly traded shares of the constituent companies, excluding shares that are not readily available for trading in the open market. This approach aims to provide a more accurate representation of the market's behavior.

Fig 3.2.1. Lyxor FTSE MIB UCITS ETF

Below you will find information about the composition of the Lyxor FTSE MIB UCITS ETF - Dist. I just represented the top 10 of the holdings out of the 41. The remaining part will be in the appendix section.

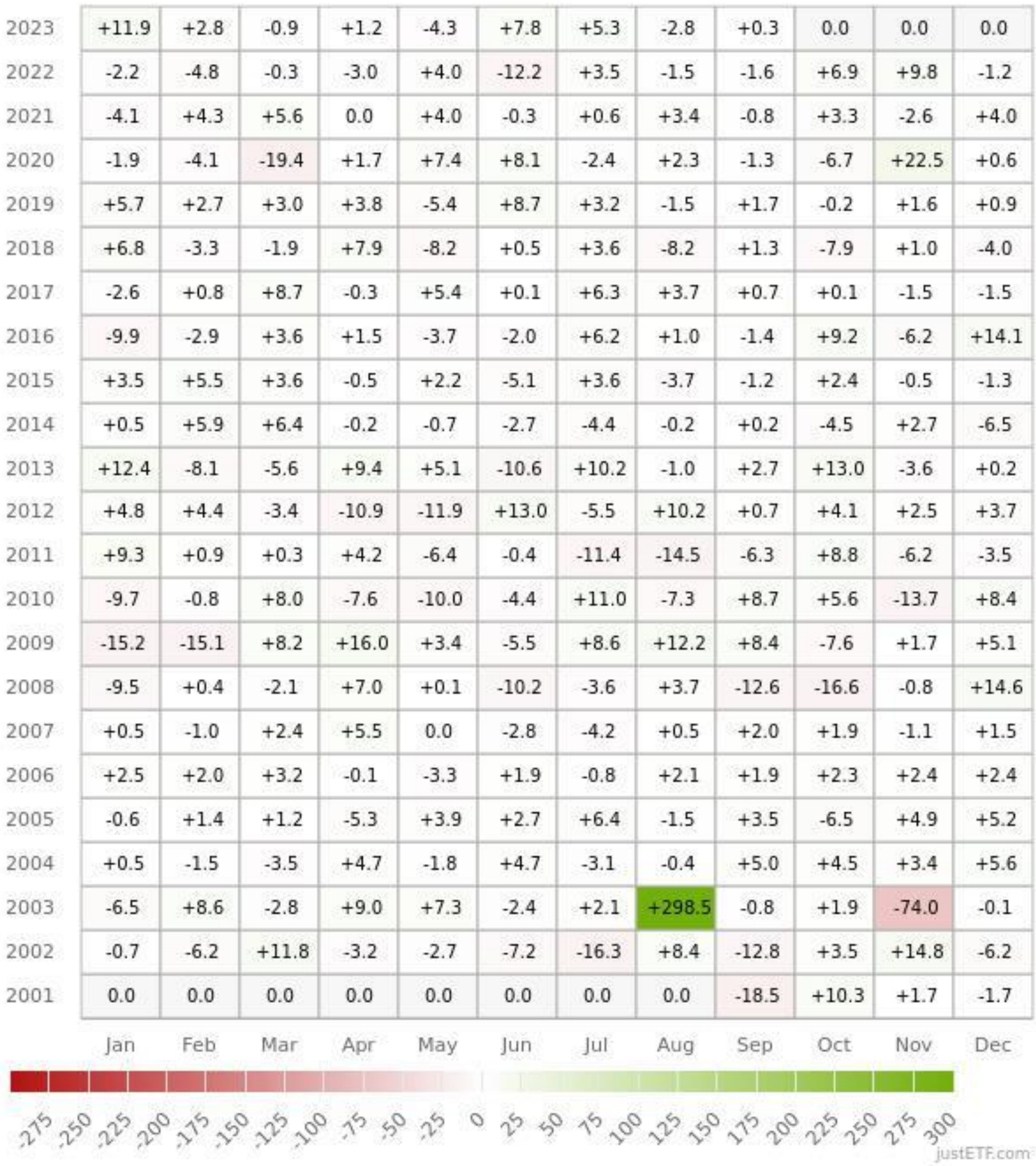
Top 10 Holdings
Weight of top 10 Holdings out of 41 70.63%

1. ENEL S.P.A. 11.03%
2. UNICREDIT SPA 9.53%
3. INTESA SAN PAOLO SPA 8.96%
4. STELLANTIS NV ORD 7.82%
5. FERRARI N.V. 7.73%
6. STMICROELECTRONICS NV 6.89%
7. ENI S.P.A. 6.83%

8. ASSICURAZIONI GENERALI S.P.A. 5.85%
9. CNH INDUSTRIAL N.V. 3.11%
10. MONCLER S.P.A. 2.88%

The ETF with the largest weighting of ENEL S.P.A. is the Xtrackers MSCI Europe Utilities ESG Screened UCITS ETF 1C, of UNICREDIT SPA is the Amundi Italy MIB ESG UCITS ETF, of INTESA SANPAOLO SPA is the Amundi Italy MIB ESG UCITS ETF. That of STELLANTIS NV ORD is the iShares STOXX Europe 600 Automobiles & Parts UCITS ETF (DE) EUR (Dist), of FERRARI N.V. is the iShares STOXX Europe 600 Automobiles & Parts UCITS ETF (DE) EUR (Dist).

The ETF with the largest weighting of STMICROELECTRONICS NV is the Xtrackers FTSE MIB UCITS ETF 1D, of ENI S.P.A. is the iShares MSCI Europe Energy Sector UCITS ETF EUR (Dist), of ASSICURAZIONI GENERALI S.P.A. is the Lyxor FTSE MIB (DR) UCITS ETF – Acc, of CNH INDUSTRIAL N.V. is the iShares Agribusiness UCITS ETF and finally that of MONCLER S.P.A. is the Amundi Italy MIB ESG UCITS ETF.



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2023	+11.9	+2.8	-0.9	+1.2	-4.3	+7.8	+5.3	-2.8	+0.3	0.0	0.0	0.0
2022	-2.2	-4.8	-0.3	-3.0	+4.0	-12.2	+3.5	-1.5	-1.6	+6.9	+9.8	-1.2
2021	-4.1	+4.3	+5.6	0.0	+4.0	-0.3	+0.6	+3.4	-0.8	+3.3	-2.6	+4.0
2020	-1.9	-4.1	-19.4	+1.7	+7.4	+8.1	-2.4	+2.3	-1.3	-6.7	+22.5	+0.6
2019	+5.7	+2.7	+3.0	+3.8	-5.4	+8.7	+3.2	-1.5	+1.7	-0.2	+1.6	+0.9
2018	+6.8	-3.3	-1.9	+7.9	-8.2	+0.5	+3.6	-8.2	+1.3	-7.9	+1.0	-4.0
2017	-2.6	+0.8	+8.7	-0.3	+5.4	+0.1	+6.3	+3.7	+0.7	+0.1	-1.5	-1.5
2016	-9.9	-2.9	+3.6	+1.5	-3.7	-2.0	+6.2	+1.0	-1.4	+9.2	-6.2	+14.1
2015	+3.5	+5.5	+3.6	-0.5	+2.2	-5.1	+3.6	-3.7	-1.2	+2.4	-0.5	-1.3
2014	+0.5	+5.9	+6.4	-0.2	-0.7	-2.7	-4.4	-0.2	+0.2	-4.5	+2.7	-6.5
2013	+12.4	-8.1	-5.6	+9.4	+5.1	-10.6	+10.2	-1.0	+2.7	+13.0	-3.6	+0.2
2012	+4.8	+4.4	-3.4	-10.9	-11.9	+13.0	-5.5	+10.2	+0.7	+4.1	+2.5	+3.7
2011	+9.3	+0.9	+0.3	+4.2	-6.4	-0.4	-11.4	-14.5	-6.3	+8.8	-6.2	-3.5
2010	-9.7	-0.8	+8.0	-7.6	-10.0	-4.4	+11.0	-7.3	+8.7	+5.6	-13.7	+8.4
2009	-15.2	-15.1	+8.2	+16.0	+3.4	-5.5	+8.6	+12.2	+8.4	-7.6	+1.7	+5.1
2008	-9.5	+0.4	-2.1	+7.0	+0.1	-10.2	-3.6	+3.7	-12.6	-16.6	-0.8	+14.6
2007	+0.5	-1.0	+2.4	+5.5	0.0	-2.8	-4.2	+0.5	+2.0	+1.9	-1.1	+1.5
2006	+2.5	+2.0	+3.2	-0.1	-3.3	+1.9	-0.8	+2.1	+1.9	+2.3	+2.4	+2.4
2005	-0.6	+1.4	+1.2	-5.3	+3.9	+2.7	+6.4	-1.5	+3.5	-6.5	+4.9	+5.2
2004	+0.5	-1.5	-3.5	+4.7	-1.8	+4.7	-3.1	-0.4	+5.0	+4.5	+3.4	+5.6
2003	-6.5	+8.6	-2.8	+9.0	+7.3	-2.4	+2.1	+298.5	-0.8	+1.9	-74.0	-0.1
2002	-0.7	-6.2	+11.8	-3.2	-2.7	-7.2	-16.3	+8.4	-12.8	+3.5	+14.8	-6.2
2001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-18.5	+10.3	+1.7	-1.7

Fig 3.2.2: Monthly returns of the ETFs in a heat map

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	-1,14%	-5,24%	-1,58%	-2,17%	2,47%	-12,85%	5,70%	-3,82%	-4,06%	9,67%	9,31%	-3,68%
2021	-2,48%	5,90%	7,84%	-1,93%	5,06%	-0,08%	1,39%	2,52%	-1,03%	5,24%	-3,42%	5,42%
2020	-0,80%	-5,39%	-22,42%	3,76%	3,63%	6,72%	-1,16%	2,82%	-3,06%	-5,66%	22,97%	1%
2019	8,00%	4,75%	3,04%	3,29%	-8,01%	7,48%	1,05%	-0,34%	3,94%	2,66%	2,64%	1,08%
2018	7,84%	-3,83%	-0,86%	7,30%	-7,87%	-0,55%	2,95%	-8,72%	2,44%	-8,03%	0,86%	-4,46%
2017	-3,14%	1,70%	8,37%	0,95%	1,88%	-0,58%	4,54%	0,86%	5,01%	0,41%	-1,70%	-2,29%
2016	-12,90%	-5,59%	2,82%	2,96%	-1,36%	-9,63%	4,03%	0,55%	-2,84%	4,41%	-0,89%	13,60%
2015	7,85%	8,89%	3,68%	-0,43%	3,30%	-4,08%	4,70%	-6,79%	-2,67%	5,35%	1,39%	-5,72%
2014	2,31%	5,30%	6,10%	0,46%	0,50%	-1,28%	-3,40%	-0,58%	2,54%	-5,34%	1,26%	-4,98%
2013	7,16%	-8,74%	-3,63%	9,55%	3,97%	-10,95%	8,12%	1,21%	4,90%	11,06%	-2%	-0,23%
2012	4,89%	3,28%	-2,26%	-8,62%	-10,46%	-11,98%	-2,65%	8,67%	0,33%	3,02%	1,91%	2,90%

Table 3.2.1 **Monthly returns of the ETFs (The Lyxor FTSE MIB UCITS ETF) reported in an excel sheet to ease the calculations.**

From this table we can clearly see the monthly returns of The Lyxor FTSE MIB UCITS ETF starting from 2012 to 2022. Looking at the historical returns we notice that there is no year that didn't get a negative return in some month. Some years like 2022 experienced great negative fluctuations in its returns. In the next step I'm going to move on calculating the average, standard deviation and variance of these data.

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	-1,14%	-5,24%	-1,58%	-2,17%	2,47%	-12,85%	5,70%	-3,82%	-4,06%	9,67%	9,31%	-3,68%
2021	-2,48%	5,90%	7,84%	-1,93%	5,06%	-0,08%	1,39%	2,52%	-1,03%	5,24%	-3,42%	5,42%
2020	-0,80%	-5,39%	-22,42%	3,76%	3,63%	6,72%	-1,16%	2,82%	-3,06%	-5,66%	22,97%	1%
2019	8,00%	4,75%	3,04%	3,29%	-8,01%	7,48%	1,05%	-0,34%	3,94%	2,66%	2,64%	1,08%
2018	7,84%	-3,83%	-0,86%	7,30%	-7,87%	-0,55%	2,95%	-8,72%	2,44%	-8,03%	0,86%	-4,46%
2017	-3,14%	1,70%	8,37%	0,95%	1,88%	-0,58%	4,54%	0,86%	5,01%	0,41%	-1,70%	-2,29%
2016	-12,90%	-5,59%	2,82%	2,96%	-1,36%	-9,63%	4,03%	0,55%	-2,84%	4,41%	-0,89%	13,60%
2015	7,85%	8,89%	3,68%	-0,43%	3,30%	-4,08%	4,70%	-6,79%	-2,67%	5,35%	1,39%	-5,72%
2014	2,31%	5,30%	6,10%	0,46%	0,50%	-1,28%	-3,40%	-0,58%	2,54%	-5,34%	1,26%	-4,98%
2013	7,16%	-8,74%	-3,63%	9,55%	3,97%	-10,95%	8,12%	1,21%	4,90%	11,06%	-2%	-0,23%
2012	4,89%	3,28%	-2,26%	-8,62%	-10,46%	11,98%	-2,65%	8,67%	0,33%	3,02%	1,91%	2,90%

Average	Variance	Std Dev
0.55	19.3	4.4

Table 3.2.2 Monthly **returns of the ETFs (The Lyxor FTSE MIB UCITS ETF) reported in an excel sheet to ease the calculations with average, variance and standard deviation. (Std Dev)**

*Bonds (Italian BTP)

The acronym identifies Polyannual Treasury Bonds, debt securities issued by the Italian government to finance public assets.

The data were collected from the website of the Bank of Italy (Banca d'Italia). These are monthly gross return of the Italian BTP (Buoni del Tesoro Poliennale). So, I collected the gross return from which I applied the tax percentage to get the monthly net returns.

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	1,35%	1,79%	1,85%	2,44%	2,99%	3,63%	3,36%	3,30%	4,14%	4,53%	4,24%	4,26%
2021	0,62%	0,59%	0,70%	0,80%	0,98%	0,87%	0,75%	0,63%	0,78%	0,95%	1,01%	1,05%
2020	1,27%	0,96%	1,55%	1,80%	1,76%	1,46%	1,20%	1,03%	0,98%	0,77%	0,66%	1%
2019	2,77%	2,81%	2,69%	2,62%	2,64%	2,28%	1,65%	1,40%	0,90%	1,00%	1,27%	1,37%
2018	1,98%	2,08%	1,97%	1,77%	2,18%	2,74%	2,64%	3,16%	2,96%	3,47%	3,39%	2,98%
2017	1,99%	2,35%	2,40%	2,26%	2,19%	2,05%	2,23%	2,11%	2,11%	2,07%	1,79%	1,79%
2016	1,53%	1,56%	1,38%	1,44%	1,53%	1,45%	1,23%	1,18%	1,27%	1,45%	1,94%	1,89%
2015	1,70%	1,56%	1,29%	1,36%	1,81%	2,20%	2,04%	1,84%	1,92%	1,70%	1,57%	1,58%
2014	3,87%	3,65%	3,40%	3,23%	3,12%	2,92%	2,79%	2,63%	2,40%	2,42%	2,29%	1,99%
2013	4,21%	4,49%	4,64%	4,28%	3,96%	4,38%	4,42%	4,42%	4,54%	4,25%	4%	4,11%
2012	6,54%	5,55%	5,05%	5,68%	5,78%	5,90%	6,00%	5,82%	5,25%	4,95%	4,85%	4,54%

Table 3.2.3 **Monthly Gross Return of the Italian BTPs**

When it comes to taxation on BTPs (Buoni del Tesoro Poliennali), there are certain tax advantages to consider. Purchasing Government Bonds, being a form of bonds, is categorized as an online trading activity and is thus subject to taxation. However, in the case of BTPs, the standard tax rate of 26% that applies to stocks, cryptocurrencies, and ETFs does not apply. Instead, a reduced tax rate of 12.5 percent is applicable. Furthermore, investing in BTPs offers the opportunity to generate income both from holding them and from selling them.

From a tax management perspective, you should refer to Article 44 of the TUIR (Consolidated Income Tax Act), under which any gains related to interest, such as dividends, bond coupons, or corporate profits, fall under the category of capital income. As such, they are subject to a substitute tax rate of 26 percent.

However, in order to encourage citizens to invest in Government Bonds, the legislator has provided for a reduction in the tax burden for those who purchase Treasury Bonds. Therefore, a reduced tax rate of 12.5 percent will apply.
The following table is the Monthly Net Returns on BTP i.e., after the tax percentage of 12.5 percent has been applied to the monthly Gross Returns.

The "gross yield" of a benchmark BTP (Buono del Tesoro Poliennale) also known as “il rendimento lordo di un BTP benchmark” refers to the annual interest rate that an investor would earn before deducting any fees or taxes. In other words, it is the actual return that the investor would receive on their bond without considering any deductions or taxes.

The term "benchmark" refers to the fact that the BTP in question is used as a reference or comparison point for the yield of other similar securities or to evaluate the performance of an investment portfolio. Benchmark BTPs are often issued by the Italian government and represent long-term government bonds with typical maturities of 10, 15, or 30 years.

When evaluating a benchmark BTP, it is important to distinguish between gross yield and net yield. Gross yield is the stated interest rate on the bond before considering taxes or other expenses. Net yield, on the other hand, takes these deductions into account and provides a more accurate estimate of what the investor will actually earn after paying taxes and fees.

In summary, the gross yield of a benchmark BTP represents the stated interest rate on the bond, which can be used for comparison and performance evaluation purposes. However, for a more comprehensive assessment of actual returns, it is important to also consider the net yield, which considers taxes and expenses.

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	1,18%	1,57%	1,62%	2,14%	2,62%	3,18%	2,94%	2,89%	3,62%	3,96%	3,71%	3,73%
2021	0,54%	0,52%	0,61%	0,70%	0,86%	0,76%	0,66%	0,55%	0,68%	0,80%	0,88%	0,92%
2020	1,11%	0,84%	1,36%	1,58%	1,54%	1,28%	1,05%	0,90%	0,86%	0,67%	0,80%	1%
2019	2,42%	2,46%	2,35%	2,29%	2,31%	2,00%	1,44%	1,23%	0,79%	0,88%	1,11%	1,20%
2018	1,73%	1,82%	1,72%	1,55%	2,91%	2,40%	2,31%	2,77%	2,59%	3,04%	2,97%	2,61%
2017	1,74%	2,06%	2,10%	1,98%	1,92%	1,79%	1,95%	1,85%	1,85%	1,81%	1,57%	1,57%
2016	1,34%	1,37%	1,21%	1,26%	1,34%	1,27%	1,08%	1,03%	1,11%	1,27%	1,70%	1,65%
2015	1,49%	1,37%	1,13%	1,19%	1,58%	1,93%	1,79%	1,61%	1,68%	1,49%	1,37%	1,38%
2014	3,39%	3,19%	2,98%	2,83%	2,73%	2,56%	2,44%	2,30%	2,10%	2,12%	2,00%	1,74%
2013	3,68%	3,93%	4,06%	3,75%	3,47%	3,83%	3,87%	4,31%	3,97%	3,72%	4%	3,60%
2012	5,72%	4,86%	4,42%	4,97%	5,06%	5,16%	5,25%	5,09%	4,59%	4,33%	4,24%	3,97%

Average	Variance	
2.2	1.6	

Table 3.2.4 **Monthly Net Return of the Italian BTPs**

From table 3.3 I applied the tax percentage of 12.5 to the values in order to get the net returns. I used the formula:

$$Net\ return = Gross\ return * (1 - 0.125)$$

Where:

0.125 is the tax percentage applied on btps i.e., 12.5%.

*CRYPOTOCURRENCY (BITCOIN)

Cryptocurrency, especially Bitcoin, is included in the portfolio because it is seen as an asset with no or low correlation with other assets as compared to the traditional assets like stocks and bonds.

The following Table shows the monthly returns of the bitcoins from 2012 to 2022.

BTC Monthly Returns Table

	January	February	March	April	May	June	July	August	September	October	November	December
2023	39.88%	0.08%	23.15%	2.96%	-7.21%	11.87%	-4.09%	-0.06%*	N/A	N/A	N/A	N/A
2022	-16.76%	12.11%	5.33%	-16.93%	-15.75%	-38.47%	19.23%	-14.35%	-2.73%	5.23%	-16.14%	-3.76%
2021	13.93%	36.01%	30.79%	-1.68%	-35.43%	-5.81%	18.72%	13.37%	-7.34%	40.16%	-7.26%	-18.75%
2020	29.37%	-8.11%	-24.92%	34.09%	9.94%	-3.35%	23.78%	3.06%	-7.72%	27.92%	42.32%	48.00%
2019	-7.06%	11.18%	7.55%	28.98%	62.02%	26.97%	-7.58%	-4.53%	-13.81%	10.65%	-17.43%	-4.78%
2018	-31.84%	2.30%	-32.58%	31.87%	-18.90%	-14.66%	21.56%	-9.18%	-5.71%	-3.91%	-37.02%	-7.91%
2017	0.18%	22.93%	-9.41%	25.19%	72.54%	6.94%	15.28%	63.79%	-8.61%	48.18%	63.40%	42.60%
2016	-15.49%	19.92%	-4.93%	7.83%	18.64%	26.38%	-7.13%	-8.33%	6.38%	14.85%	6.27%	30.20%
2015	-31.31%	16.32%	-4.07%	-2.90%	-2.77%	14.25%	8.11%	-18.80%	2.59%	32.30%	21.13%	13.98%
2014	8.31%	-30.65%	-19.36%	-2.71%	39.44%	2.78%	-8.58%	-17.69%	-19.31%	-13.11%	11.85%	-15.70%
2013	57.69%	56.32%	176.12%	46.27%	-5.41%	-25.47%	15.67%	24.85%	-4.44%	54.87%	453.00%	-33.02%
2012	13.11%	-13.27%	1.02%	1.01%	3.80%	28.96%	41.04%	14.79%	15.13%	-10.70%	13.55%	7.43%

Fig 3.2.3: Monthly return of Bitcoins. Source: Into the cryptoverse

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	-16,76%	12,11%	5,33%	-16,93%	-15,75%	-38,47%	19,23%	-14,35%	-2,73%	5,23%	-16,14%	-3,76%
2021	13,93%	36,01%	30,79%	-1,68%	-35,43%	-5,81%	18,72%	13,37%	-7,34%	40,16%	-7,26%	-18,75%
2020	29,37%	-8,11%	-24,92%	34,09%	9,94%	-3,35%	23,78%	3,06%	-7,72%	27,92%	42,32%	48%
2019	-7,06%	11,18%	7,55%	28,98%	62,02%	26,97%	-7,58%	-4,53%	-13,91%	10,65%	-17,43%	-4,78%
2018	-31,84%	2,30%	-32,58%	31,87%	-18,90%	-14,66%	21,55%	-9,18%	-5,71%	-3,91%	-37,02%	-7,91%
2017	0,18%	22,93%	-9,41%	25,19%	72,54%	6,99%	15,28%	63,79%	-8,61%	48,18%	63,40%	42,60%
2016	-15,49%	19,92%	-4,93%	7,83%	18,64%	26,38%	-7,13%	-8,33%	6,38%	14,85%	6,27%	30,20%
2015	-31,31%	16,32%	-4,07%	-2,90%	-2,77%	14,25%	8,11%	-18,80%	2,59%	32,30%	21,13%	13,98%
2014	8,31%	-30,65%	-19,36%	-2,71%	39,44%	2,78%	-8,56%	-17,69%	-19,31%	-13,11%	11,85%	-15,70%
2013	57,69%	56,32%	56,32%	46,27%	-5,41%	-25,47%	15,67%	24,85%	-4,44%	54,87%	453%	-33,02%
2012	13,11%	-13,27%	-13,27%	1,01%	3,80%	28,96%	41,04%	14,79%	15,13%	-10,70%	13,55%	7,43%

Fig 3.2.5 Monthly return of Bitcoins in an excels sheet.

In the next table it's possible to notice the average and variance of the bitcoins calculated using excel.

	January	Febuary	March	April	May	June	July	August	September	October	November	December
2022	-16,76%	12,11%	5,33%	-16,93%	-15,75%	-38,47%	19,23%	-14,35%	-2,73%	5,23%	-16,14%	-3,76%
2021	13,93%	36,01%	30,79%	-1,68%	-35,43%	-5,81%	18,72%	13,37%	-7,34%	40,16%	-7,26%	-18,75%
2020	29,37%	-8,11%	-24,92%	34,09%	9,94%	-3,35%	23,78%	3,06%	-7,72%	27,92%	42,32%	48%
2019	-7,06%	11,18%	7,55%	28,98%	62,02%	26,97%	-7,58%	-4,53%	-13,91%	10,65%	-17,43%	-4,78%
2018	-31,84%	2,30%	-32,58%	31,87%	-18,90%	-14,66%	21,55%	-9,18%	-5,71%	-3,91%	-37,02%	-7,91%
2017	0,18%	22,93%	-9,41%	25,19%	72,54%	6,99%	15,28%	63,79%	-8,61%	48,18%	63,40%	42,60%
2016	-15,49%	19,92%	-4,93%	7,83%	18,64%	26,38%	-7,13%	-8,33%	6,38%	14,85%	6,27%	30,20%

2015	-31,31%	16,32%	-4,07%	-2,90%	-2,77%	14,25%	8,11%	-18,80%	2,59%	32,30%	21,13%	13,98%
2014	8,31%	-30,65%	-19,36%	-2,71%	39,44%	2,78%	-8,56%	-17,69%	-19,31%	-13,11%	11,85%	-15,70%
2013	57,69%	56,32%	56,32%	46,27%	-5,41%	-25,47%	15,67%	24,85%	-4,44%	54,87%	453%	-33,02%
2012	13,11%	-13,27%	-13,27%	1,01%	3,80%	28,96%	41,04%	14,79%	15,13%	-10,70%	13,55%	7,43%

Average	Variance
7.7	1138.5

Fig 3.2.6 Monthly returns of the Bitcoins with variance and average

COMPUTATIONS

*Expected Return /Average ($\bar{x}$):

The expected return of an asset is the anticipated or estimated return that an investor can expect to earn from that asset over a specific future period. It represents the average or mean return an investor might receive.

This computation was also done in excel using the following formula for each asset.

$$\bar{x} = \frac{\Sigma r_i}{N}$$

Where:

$\bar{x}$ is the average.

r_i is the value of the i-th element.

N is the sample size.

Applying this formula in excel I came up with the following expected return for each asset.

ETFs = 0.55

BTP = 2.2

Bitcoins = 7.7

The averages imply:

The average return for ETFs of 0.55 means that, on average, ETF investments have provided a return of 0.55% over a specific period. Investors who hold ETFs in their portfolios have experienced, on average, this level of return from their ETF investments during that period.

The average return for BTPs of 2.2% suggests that, on average, investors who hold BTPs have earned a return of 2.2% on their investments over a specific time frame. This return represents the yield or interest income generated by BTPs.

The average return for Bitcoins of 7.7 indicates that, on average, the returns on Bitcoin investments have been 7.7% over a particular time period. Bitcoin is a highly volatile cryptocurrency, and its average return of 7.7% suggests that it has experienced significant price fluctuations over the specified time frame.

These average returns provide insights into the historical performance of these assets over a specific period. Investors use such information to assess the past performance of assets and make informed investment decisions based on their risk tolerance and investment objectives.

- Variances (σ^2): The variance represents the average of the squared differences between each return and the mean return. It's a measure of the asset's volatility, a higher variance indicates greater volatility and vice versa. It's calculated as follows:

$$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

Where:

σ^2 is the variance of each asset.

x_i is the value of the i-th element.

$\bar{x}$ is the mean value.

n is the number of the observation / sample size.

After applying this formula in Microsoft excel, I came up with the following variances for EYFs, BTP and Bitcoins respectively: 19,3 ;1,6; and 1138,5.

It represents the variance of returns for each of these assets. Variance is a measure of the dispersion or spread of returns around the mean (average) return for an asset. Thes variance values imply:

- The ETFs have a variance of 19.3, indicating that the returns of ETFs are relatively spread out or volatile. In other words, the returns have a significant degree of variability from the average return. This suggests that ETFs may have a moderate level of risk or price fluctuation, but it's not extremely volatile compared to other assets.
- The BTP (Buono del Tesoro Poliennale) has a variance of 1.6, indicating that the returns of BTP are relatively less spread out or less volatile compared to ETFs. It suggests that BTP may have a lower level of risk or price fluctuation compared to ETFs, making it a potentially more stable investment.
- Finally, Bitcoins have a very high variance of 1138.5, indicating that the returns of Bitcoins are highly spread out and extremely volatile. This suggests that Bitcoins are associated with a very high level of risk and price fluctuation. It's considered a highly speculative and volatile asset.

Variance provides insights into the risk or volatility of an asset's returns. A higher variance indicates greater volatility and risk, while a lower variance implies less volatility and risk. Therefore, based on these variances' values, Bitcoins are the riskiest and most volatile among the three assets, followed by ETFs, while BTP is the least volatile.

- Covariance (Cov):

Covariance between assets measures how two or more assets move together or relate to each other in terms of their returns. It provides insights into the degree to which the returns of one asset tend to move in relation to the returns of another asset. It can be calculated as follows:

$$\text{cov}(x, y, z) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})(z_i - \bar{z})}{N - 1}$$

Where:

$\text{cov}(x, y, z)$:is the covariance between assets X, Y, and Z.

x_i, y_i, z_i : are e returns of assets X, Y, and Z for the i-th observation or time period.

$\overline{x}, \overline{y}, \bar{z}$: are the mean returns of assets X, Y, and Z, respectively.

N: is the number of observations or time periods.

After the computation in excel I got the values of the assets to be:

-Covariance between ETFs and BTPs = 0.06

- Covariance between ETFs and Bitcoins = 11.02

- Covariance between ETFs and Bitcoins = 3.63

Comments:

First Case: A covariance of 0.06 indicates that there is a positive relationship between the returns of ETFs and BTPs, but it is a relatively weak positive relationship.

The positive sign of the covariance indicates that when the returns of ETFs are above their average (mean) returns, the returns of BTPs tend to also be above their average returns. Similarly, when the returns of ETFs are below their average returns, the returns of BTPs tend to be below their average returns. In other words, they move in the same general direction, but the relationship is not extremely strong.

The magnitude of the covariance (0.06) indicates the strength of the relationship. A covariance of 0.06 is relatively small, suggesting that while there is a tendency for ETF and BTP returns to move together, the degree of co-movement is not very high. This means that changes in the returns of ETFs are not highly predictive of corresponding changes in BTP returns and vice versa.
From a portfolio perspective, having assets with a positive but relatively low covariance can be beneficial. It means that these assets do not move in lockstep, and when combined in a portfolio, they can potentially provide some level of risk diversification. If one asset performs poorly, the other may help offset the losses to some extent.

Therefore, a covariance of 0.06 between ETFs and BTPs suggests a modest positive relationship between their returns, indicating that they tend to move in the same direction to some extent, but the relationship is not very strong.
Second Case:
A covariance of 11.0 between ETF and Bitcoin indicates that there is a positive relationship between the returns of ETFs and Bitcoin, and it is a relatively strong positive relationship. The positive sign of the covariance indicates that when the returns of ETFs are above their average (mean) returns, the returns of Bitcoin tend to also be above their average returns. Similarly, when the returns of ETFs are below their average returns, the returns of Bitcoin tend to be below their average returns. In other words, they move in the same general direction, and the relationship is relatively strong. The magnitude of the covariance (11.0) indicates the strength of the relationship. A covariance of 11.0 is relatively large, suggesting that ETF and Bitcoin returns tend to co-move significantly. Changes in the returns of ETFs are likely to be accompanied by corresponding changes in Bitcoin returns to a substantial degree.
From a portfolio perspective, having assets with a positive covariance means that they are positively correlated. This implies that when building a portfolio that includes both ETFs and Bitcoin, the movements in their returns will tend to reinforce each other. If one asset performs well, the other is likely to do so as well, but if one underperforms, the other is also likely to underperform.

So, a covariance of 11.0 between ETFs and Bitcoin suggests a strong positive relationship between their returns, indicating that they tend to move together in a relatively strong and coordinated manner. This can be important information for portfolio construction and risk management.

Third Case :

A covariance of 3.63 between BTP (Buoni del Tesoro Poliennali) and Bitcoin implies a positive relationship in their returns, but it's relatively weak. This positive covariance sign indicates that when BTP returns are above their average, Bitcoin returns tend to be above their average as well, and vice versa. While they generally move in the same direction, the strength of this relationship is not substantial, as evidenced by the relatively small covariance value.

The magnitude of the covariance (3.63) suggests that although there is a tendency for BTP and Bitcoin returns to move together, they are not strongly correlated. This means that changes in BTP returns are not highly indicative of corresponding changes in Bitcoin returns and vice versa.

From a portfolio perspective, this positive but relatively low covariance can be advantageous. It means that these assets do not move perfectly in sync, potentially offering some diversification benefits when combined in a portfolio. If one asset underperforms, the other may help mitigate losses to some extent.

In summary, a covariance of 3.63 between BTP and Bitcoin indicates a modest positive relationship in their returns, implying that they generally move in the same direction, but the relationship is not particularly strong.

- Standard Deviation

It's a critical measure of risk and volatility. It provides insights into how the individual assets within the portfolio interact with each other and how the overall portfolio's risk is affected by their combinations.

It quantifies the total risk of the portfolio, considering not only the risk of each individual asset but also their correlations or relationships. A higher portfolio standard deviation indicates a riskier and more volatile portfolio, while a lower standard deviation suggests a less risky and more stable portfolio.

It's calculated as follows:

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}}$$

σ = is the standard deviation.

μ = is the mean.

N = is the sample size.

x_i = is the value of the one observation.

From the computation in excel using this formula I get the following standard deviation for each asset in the portfolio.

ETFS = 4.4

BTP = 1.3

Bitcoins = 33.7

Looking at these standard deviations, we can clearly see that Bitcoins have the highest standard deviation (33.7), indicating that they are the most volatile and risky among the three assets. ETFs have a moderate level of risk (4.4), while BTPs are the least risky (1.3).

A higher standard deviation suggests that an asset's returns are more spread out from the mean (average), indicating greater price fluctuations over time. Therefore, Bitcoins are subject to significant price swings, which can lead to both substantial gains and losses. ETFs exhibit moderate volatility, and BTPs have relatively stable returns.

Until now we have seen the statistical measures of the single asset in the portfolio. But what about the entire portfolio? To answer this question, we are going to see the expected return and the variance of the entire portfolio.

- Expected Return of the entire portfolio; $E(rp)$:

$$E(rp) = \sum_{i=1}^{n} \omega_i E(r_i)$$

$E(rp)$ = the expected return of the portfolio

ω_i = are the weights given to each asset.

$E(r_i)$ = is the expected average of each asset

- Variance of the portfolio (σ_p^2)

  It's calculated as follows:

$$(\sigma_p^2) = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{i \neq 1} \omega_i \omega_j \sigma ij$$

$$= \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + \omega_3^2\sigma_3^2 + 2\omega_1\omega_2 Cov(1,2) + 2\omega_1\omega_3 Cov(1,3) + 2\omega_2\omega_3 Cov(2,3)$$

Where:

$\sigma_1^2, \sigma_2^2, \sigma_3^2$ = *are the individual variances.*

$\omega_1, \omega_2, \omega_3$ = *are the weights of each asset in the portfolio.*

$Cov(1,2)$, $Cov(1,3)$, $Cov(2,3)$ = *are the covariances between the different assets.*

We now have all the necessary elements in order to construct our Covariance-Variance Matrix.

Variance-Covariance Matrix.

To construct this matrix, we need the variance and covariance of asset 1, asset2 and asset3.

Let the asset be denoted as follows:

Let A be the matrix of the variance-covariance of the assets in the portfolio.

σ_{11}^2 = *Variance of ETF*

σ_{22}^2 = *Variance of BTP*

σ_{33}^2 = *Variance of Bitcoins*

σ_{12} *or* σ_{21} = *Covariance between ETF and BTP*

σ_{13} or σ_{31} = Covariance between ETF and Bitcoins

σ_{23} or σ_{32} = Covariance between BTP and Bitcoins

In the main diagonal of the matrix, we have the differences variance and in the upper and lower diagonal we have the different covariances.

$$A = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}^2 \end{bmatrix}$$

Therefore, substituting the above matrix with the exact values we have the following matrix:

$$\mathrm{A} = \begin{bmatrix} 19.3 & 0.06 & 11 \\ 0.06 & 1.6 & 3.6 \\ 11 & 3.6 & 1138.5 \end{bmatrix}$$

Finally, at this given stage it's possible to proceed with the computation of the optimization problem using the Markowitz approach.
The objective of the maximization problem in the Markowitz theory is to find the amount of wealth (in percentage) we need to invest in the different risky assets in our portfolio ie., (ETFs, BTP and Bitcoins), these are all risky assets. After getting each percentage we will invest the rest of our wealth in risk-free assets (on average they yield 1%).
In order words, the objective of the maximization problem in the Markowitz model, also known as the Markowitz Portfolio Optimization model, is to find the optimal allocation of assets in a portfolio to maximize the portfolio's expected return while simultaneously minimizing its risk or volatility. So, the goal is to construct a portfolio that provides the highest possible return for a given level of risk or the lowest possible risk for a given level of return. This optimization problem aims to strike a balance between risk and return, allowing investors to make rational investment decisions based on their risk tolerance and return objectives.

To proceed we will do some simple assumptions:
Let suppose the initial wealth is = 1
Ŷ is the return of the asset
Eŷ is the expected return of the asset.
Var Ŷ is the variance of the portfolio.

Therefore,

$$\hat{y} = 1 + rf\left(1 - \sum_{1}^{n} ia_i\right) + \sum_{1}^{n} ai\bar{y}i$$

$$\mathrm{E}\hat{y} = 1 + rf\left(1 - \sum_{1}^{n} ia_i\right) + \sum_{1}^{n} ia_i\,E(\bar{y}i)$$

$$Var\hat{y} = \sum_{1}^{n} i \sum_{1}^{n} j \ ai\ aj\sigma ij$$

So, the Markowitz approach is just to maximize the following:

Max $E\hat{y} - \frac{1}{2} A\, Var\hat{y}$

Maximizing the above linear function means maximizing the average portfolio i.e.,

MAX : $1 + rf(1 - \sum_1^n ia_i) + \sum_1^n ia_i\, E(\bar{y}i) - \frac{1}{2} A \sum_1^n i \sum_1^n j \ ai\ aj\sigma ij$ *(1)*

The first order condition (FOC) of this equation is given by:

FOC: $E(\hat{y}i) - rf = A.\sum_1^n j\, a_j\, \sigma ij$ *(2)*

rf is the risk-free asset and is assumed to be 1%

Now let's find the amount of wealth to invest in each risky asset.

COMPUTATIONS:

Number of risky assets = 3

$\sigma_{11}^2 = 19.3$ $\sigma_{22}^2 = 1.6$ $\sigma_{33}^2 = 1138.5$

$\sigma_{12} = 0.06$ $\sigma_{13} = 11$ $\sigma_{23} = 3.6$

$a_1\ a_2\ a_3$

$$
\begin{cases}
E(\hat{y}1) - rf = A(a_1\,\sigma_{11}^2 + a_2\,\sigma_{12} + a_3\,\sigma_{13)} \\
E(\hat{y}2) - rf = A(a_1\,\sigma_{21}^2 + a_2\,\sigma_{22}^2 + a_3\,\sigma_{23}) \\
E(\hat{y}3) - rf = A(a_1\,\sigma_{31}^2 + a_2\sigma_{32} + a_3\,\sigma_{33)}^2
\end{cases}
$$

↙

$$
\begin{cases}
E(\hat{y}1) - rf = A(19.3a_1 + 0.06a_2 + 11a_3) \\
E(\hat{y}2) - rf = A(0.06a_1 + 1.6a_2 + 3.6a_3) \\
E(\hat{y}3) - rf = A(11a_1 + 3.6a_2 + 1138.5a_3)
\end{cases}
$$

↙

$$
\begin{cases}
E(\hat{y}1) - rf = 19.3Aa_1 + 0.06Aa_2 + 11Aa_3 \\
E(\hat{y}2) - rf = 0.06Aa_1 + 1.6Aa_2 + 3.6Aa_3 \\
E(\hat{y}3) - rf = 11a_1 + 3.6Aa_2 + 1138.5Aa_3
\end{cases}
$$

- a_3

$$E(\hat{y}3) - rf = 11a_1 + 3.6Aa_2 + 1138.5Aa_3$$

$=11A\{1/19.3(E(\hat{y}3) - rf - 0.06a_2 - 11Aa_3)\} + 3.6A\{1/1.6(E(\hat{y}2) - rf - 0.06Aa_1 - 3.6Aa_3)\} + 1138.5c$

$=\{0.57E(\hat{y}1) - rf - 0.034\, a_2 - 6.27A\, a_3\} + \{2.25E(\hat{y}2) - rf - 0.135a_1 - 7.4Aa_3\} + 1146.1Aa_3$

$=E(\hat{y}3) - rf = 0.57\, E(\hat{y}1) - rf + 2.25E(\hat{y}2) - rf - 0.135a_1 - 1146.2A$

$=E(\hat{y}3) - rf - 0.57\, E(\hat{y}1) - rf - 2.25E(\hat{y}2) - rf = \frac{1146.2Aa_3}{1146.2A}$

$=\frac{1}{-1146.2}\{[E(\hat{y}3) - rf] + 0.57[E(\hat{y}1) - rf] + 2.25[E(\hat{y}2) - rf]\}$

$=\frac{1}{1146.2}[(2.2) + 0.57(0.55) + 2.25(7.7)]$

$=0.0019+0.3135+17.325$

$a_3 = 17.64\%$

- a_1:

$$E(\hat{y}1) - rf = 19.3Aa_1 + 0.06Aa_2 + 11Aa_3$$

$$a_1: E(\hat{y}1) - rf - 0.06Aa_2 - 11Aa_3 = \frac{19.3Aa_1}{19.3A}$$

$$a_1 = \frac{1}{19.3}[E(\hat{y}1) - rf - 0.06a_2 - 11a_3]$$

$$a_1 = \frac{1}{19.3}(0.55) - 0.06a_2 - 11a_3$$

$$=1/19.3\,[(0.55) + 0.06\,(13.06) + 11\,(17.64)]$$

$$=0.028 + 0.04 + 10.05$$

$$a_1 = 10.06\%$$

- a_2

$$E(\hat{y}2) - rf = 0.06Aa_1 + 1.6Aa_2 + 3.6Aa_3$$

$$E(\hat{y}2) - rf - 0.06Aa_1 - 3.6Aa_3 = \frac{1.6Aa_3}{1.6A}$$

$$a_2 = \frac{1}{1.6} \quad [E(\hat{y}2) - rf - 0.06a_1 - 3.6a_3]$$

$a_2 = \frac{1}{1.6}$ *(7.7)* $- 0.06a_1 - 3.6a_3$

=4.812-0.06(0.028-0.06 $a_2 - 194) - 63.5$

=4.812-0.00168+0.0036 $a_2 + 11.64 - 63.5$

=-47.04+0.0036 a_2

$$\frac{47.04}{0.0036} = \frac{0.0036\, a_2}{0.0036}$$

$a_2 =$ *13.06%*

Therefore, the percentual amount of the wealth is divided like this:
10.06% in ETFs ,13.06% in BTP and 17.64% in Bitcoins. This means that 40.7% of the wealth will be invested in risky assets and 59.24 % in risk free assets.

CHAPTER IV: CRITICAL ANALYSIS OF FINDINGS

4.1 Discussion

Looking at the different results obtained we can comment on many aspects: Portfolio Considerations: These figures can be used in portfolio construction. Combining assets with different expected returns and standard deviations can help optimize risk-return trade-offs.

- ETFs and BTPs, with their lower volatility and moderate expected returns, could serve as stabilizing components in a portfolio.

- Bitcoins, with their higher expected return but also significantly higher risk, could potentially provide a portfolio with growth opportunities but would increase overall portfolio risk.

CONCLUSION

In this paper, we conducted an in-depth examination of the Markowitz model and its applications in the field of portfolio management and optimization. We delved into the underlying theories, including the concepts of the efficient frontier, diversification, and the trade-off between risk and return.

Throughout our research, we arrived at several significant aspects

- We demonstrated how the Markowitz model empowers investors to construct efficient portfolios that either maximize expected returns for a given level of risk or minimize risk for a target level of expected returns. This fundamental concept underscores the essence of portfolio optimization.
- We underscored the critical importance of diversification in reducing the overall risk of a portfolio. Diversification involves selecting a mix of assets with relatively low correlations, as it plays a pivotal role in mitigating risk. By holding assets that do not move in lockstep, investors can achieve a more balanced risk-return profile.
- We critically examined the limitations of the Markowitz model, including its reliance on the assumption of a normal distribution of returns and its disregard for behavioral factors that can influence market dynamics. While the model provides valuable insights, it's essential to acknowledge these constraints and consider their implications in real-world scenarios.

Based on the different results obtained and the principles of the Markowitz theory, we can draw several conclusions:

- Risk and Return Profile: The assets in our portfolio have significantly different risk and return profiles. Bitcoins have the highest expected return (7.7%) but also the highest standard deviation (33.7), indicating both the potential for high returns and high volatility. BTPs, on the other hand, have a lower expected return (2.2%) and much lower volatility (1.3), making them a relatively safer but lower-yielding investment. ETFs fall in between, with an expected return of 0.55% and a standard deviation of 4.4.

- Diversification: Diversification plays a crucial role in portfolio optimization. By investing in assets with low correlations, you can potentially reduce overall portfolio risk. In this case, the percentages of income invested in each asset suggest some level of diversification, with no single asset dominating the portfolio.

- Efficient Frontier: The Markowitz theory allows us to construct portfolios along the efficient frontier, which represents the highest expected return for a given level of risk or the lowest risk for a given level of expected return. To make the best use of the Markowitz model, you can calculate the expected return and standard deviation of the entire portfolio based on the weights assigned to each asset.

- Risk Tolerance: The choice of asset allocation should align with your risk tolerance and investment goals. A portfolio with a higher allocation to Bitcoins is riskier but has the potential for higher returns, while a portfolio with a higher allocation to BTPs is less risky but may yield lower returns.

- Optimization: The Markowitz theory provides a framework for optimizing portfolio allocation based on risk and return preferences. Depending on your risk tolerance and objectives, you can adjust the weights of each asset to achieve your desired risk-return trade-off.

It's important to note that constructing a portfolio involves more than just these statistics. Diversification, correlation among assets, investment goals, and risk tolerance are also critical factors to consider.

Additionally, these figures are based on historical data and market conditions, which can change over time. Therefore, regular portfolio review and adjustment are necessary to adapt to changing market dynamics and investor preferences.

The implications of our findings are of paramount importance to investors and financial professionals. The ability to construct efficient portfolios based on the principles of Markowitz can empower investors to optimize their risk-return trade-offs, making more informed decisions about managing their wealth.

Despite the numerous contributions of the Markowitz model, there remain unexplored research avenues. Future studies could delve deeper into evaluating the model's assumptions, exploring novel methods for risk management, and integrating environmental, social, and governance (ESG) factors into the portfolio optimization process.

Therefore, to wrap up, our portfolio reflects a diversified approach with different asset classes, considering the Markowitz theory. The specific allocation to each asset should be based on each individual risk tolerance, investment horizon, and financial goals. It’s important to note that the efficient frontier and optimal asset allocation can vary for each investor, so it's essential to tailor the portfolio to your unique circumstances. Additionally, regular portfolio monitoring and rebalancing are crucial to maintain the desired risk-return profile over time.

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