A Numerical-Regression-Based, Weight-Optimized Reliable Method for Ship and Performance Vessels Structures Scantling in Different Composite Materials

Un Metodo Numerico Affidabile di Dimensionamento Strutturale Ottimizzato al Peso Globale per Navi e Barche Performanti in Diversi Materiali Compositi.

Master of science (MSc) in Ocean and Marine Engineering & Naval Architecture
Master's Thesis

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To my beloved parents
Simona and Alessandro

Contents

List of Figures

List of Tables

Abstract

In the spirit of research, a very convenient mean to understand physic mechanics, improve available technologies or alternatively enrich our scientific knowledge, is to interpose new problems yet to be solved for which these are their solutions. The spirit behind this Master's Thesis to be developed is to see whether is effectively feasible to combine this philosophy with one of the most relevant aspects in the Marine industry: ship structures.

Not only in the Naval Architecture world, but also in a whole variety of Industrial Engineering fields, structural problems are considered relevantly some of the most challenging enigmas of an asset design; this because while fulfilling defined rigidity requirements, assets are generally asked to perform complying to the owner weight needs. A ship, for example, should be able to transport a large amount of cargo across oceans while consuming as less as possible in complete safety. Racing vessels else way should be as fast as possible but be capable to resist to large sea-state loads. On this lines, meeting performances and requirements together isn't always easily achievable and represents one of the main challenges behind this Master's Thesis.

The requirements environment in which a ship or a vessel may sail has become increasingly mathematically complex at the point that its structure must be defined according to specific therms collected within the *Classification Societies Rules* which can be always downgraded if proven the structure safety. On the other hand, performances are represented by a set of different attributes from the owner side that the asset is supposed to fulfill appropriately without any exception. Along this logical thread, whether possible, Engineers shall wisely profit from the regulations downgrading advantage to develop new procedures and reliable methods to properly weight-optimize structures, and always gain deign margins to create an asset which performs properly and according to the client requests.

To accomplish this task, designers shall combine this design philosophy together with the available technologies such as numerical problems resolution applications. The present Master of Science thesis aims to present a reliable numerical method for ship structures scantling and weight-optimization, capable of providing as quick as possible the most suitable, performing and safe layout.

Despite this, the student would appreciate to warn the reader that the followed procedure might not suit to any possible design, but at the same time that this can be quite easily adjusted and refined to the project.

The next chapters will overview the developed procedure and will particularly detail some of the many encountered issues and how these have been properly fixed.

Chapter 1

Introduction to composite materials

When processing a structural design, the material choice surely plays a crucial point of the whole project. In some cases this happens to be imposed by the owner or the shipyard production availability, in others instead the past shipbuilding know-how provides different layout solutions and categories of materials depending on the purpose of use.

Whenever none of these options result to be possible, or the shipyard is asked to make a full research to find the best compromise between weight, costs and feasibility, a different approach is adopted by performing a series of tests and considerations in order to find the most suitable material for the project.

As far as the ship structures are concerned these may be categorized in three main classes: metals, woods and composites. These present different stiffness, weights and costs and are hardly limited in their use by the manufacturer know-how and machinery availability. Generally speaking metals like aluminum or steel are used on large scale projects, whenever global loads (and hull girder stiffness) are the main concern, while composites material for smaller vessels (up to 170 feet lengths or so) where lightness and speeds might be the most relevant aspects. Despite in fact these may result actually resistant enough, longer composite hulls would be subject to much larger displacements, resulting in an overall unreliable structure.

This being said, since these type of materials are not part of the Naval Architecture and Marine Engineering course of study, a brief overview over composites is required before even approaching the newly proposed design method.
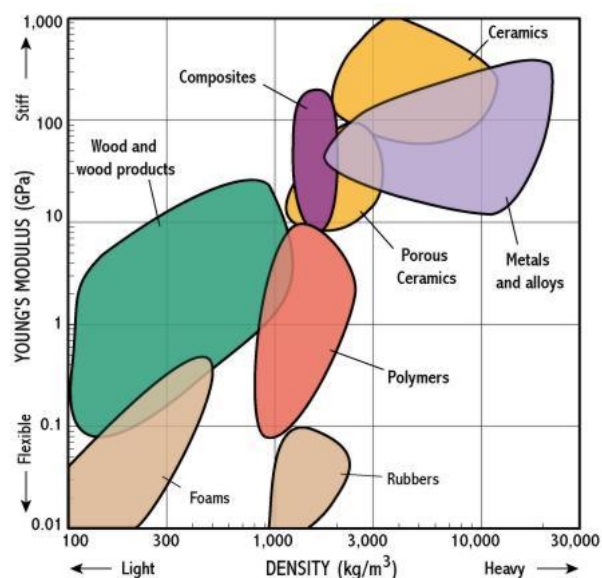
Figure 1.1: Different categories of materials in their stiffness to density range

1.1 Composite structure background

From the 1950s to the 1990s, advances in materials and fabrication techniques used in the pleasure craft industry have helped to reduce production costs and improve product quality, making composites some of the most innovative and studied construction techniques existing nowadays.

In the early days, composite laminates where produced in single skin structures, namely a mix of two materials differing by chemical composition and mechanical properties: a reinforcement and a matrix. Around 1970s, designers realized that increasingly stiffer and lighter structures could be achieved in the end if a sandwich construction technique was adopted. Laminating in fact an inner and outer skin to a low density core, the fibers where located at a greater distance from the panel's neutral axis, performing exceptionally well when subjected to bending loads produced by hydrodynamic forces.

1.1.1 Reinforcements

Reinforcements are generally made out by overlays of different (or equal) sheets of fibers. These may be distinguished for their chemical composition, weight, cured strength and orientation.

Typically used materials in the marine industry are Fiberglass, Carbon and Aramid fibers (*Kevlar*). There isn't a specific type of fiber which is better than the others since they differ generally for their weight, mechanical properties and prices, but they can be easily chosen to suit the task they are used for.

Commonly, carbon fibers are used where a very high structural stiffness is needed thanks to their high Young modulus values (masts or very long light hulls), in particular where costs are not a main concern due to its very high price. This is why, despite being feasible, merchant ships can't be built out of composites without being subject either to very high displacements or ridiculous costs. Softer composites such as fiberglass and Kevlar are instead used where strength and weight might not be a major aspect for which it is preferable to keep contained costs (high scale production of small vessels). Besides, for appendixes highly subjected to damages is quite frequent the use of Aramid combined either with fiberglass or carbon fibers, thanks to their feature that do not allow cracks to shatter when damaged by collisions or groundings (rudders and keels).

At the same time almost any of the above are weaved to create different type of fabrics used to increase the strength in different directions. It is typically common to distinguish continuous fibers into uni- or bi-directional (on the same sheet), in order to identify if fibers are oriented into only one or two different directions (figure 1.2). Discontinuous fibers instead can be made into directional or random direction filaments (chopped strand mat).
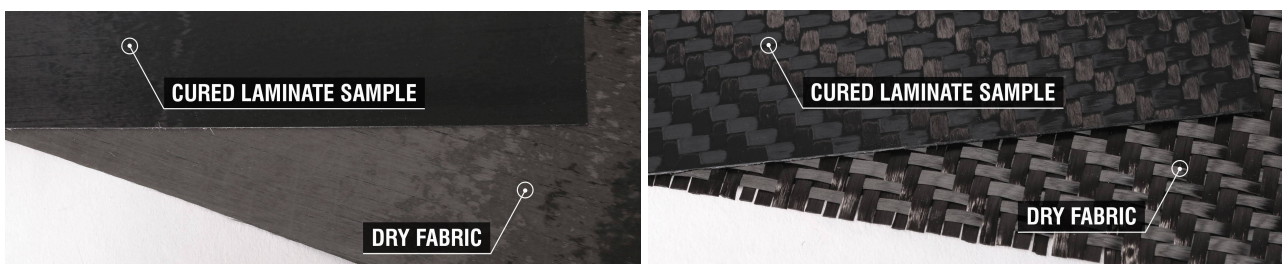
Figure 1.2: Uni- (left) and Bi-directional (right) carbon fibers in dry fabric and cured versions

1.1.2 The matrix

The matrix can be either a thermoplastic or a thermoset product. The first one, rarely used in naval constructions, turns into liquid when heated and freezes into a very glassy state if cooled sufficiently. The second instead solidifies by the application of heat or through a chemical reaction, which makes its cure

irreversible. Unfortunately, thermoplastic matrices still do not meet the minimum structural requirements, hence their use is limited to nonstructural components. This fact poses serious issues regarding recyclability of FRP materials, as thermoset matrices cannot be separated by the fibers at the end of the life cycle, unless by energy consuming and often highly polluting complex chemical processes.

Thermoset matrix can be either polyester or epoxy: polyester matrix is supplied as a solution of unsaturated polyester in styrene, where the curing process starts when a catalyst is added to the solution in minor quantity. On the other hand, the epoxy matrix is generally supplied by two liquid components, namely named resin and hardener, where the curing process begins when the two components are mixed together.

The main purposes of the matrix are to distribute loads uniformly among the fibers and offer interlaminar shear strength. The dry fibers in fact present themselves as a common fabric which is not sufficient to create a fixed structure. These aspects may differ from type to type with particular reference to their chemical compositions.

In marine structures, the two most used resins are to be considered the Polyester and the Epoxy ones. The firsts are the simplest and most affordable, offering higher chemical resistance but major levels of toxicity. These generally consists of unsaturated material, such as maleic anhydride or fumaric acid, dissolved in a reactive monomer, such as Styrene. The rigidity of polyester resins can be lessened by increasing the ratio of saturated to unsaturated acids at the manufacturer concern. Flexible resins may be advantageous for increased impact resistance, however, this comes at the expense of the overall stiffness. Nonstructural laminate plies, such as gelcoats and barrier veils, are sometimes formulated with more flexible resins to resist local cracking. On the other hand of the spectrum, low-profile resins are designed to minimize reinforcement print-through.

Table 1.1: Comparison between Polyester and Epoxy resins

Composition	
Polyester resin is produced by free-radical polymerization	Epoxy resin is produced by condensation polymerization
Adhesive properties	
Polyester resins have lower interlaminar strength	Epoxy resins have higher adhesive properties
Shrinkage	
High shrinkage	Low shrinkage
Environmental durability	
High environmental durability	Low environmental durability
Applications and use	
Polyester resins are less likely to be used in high thermal applications, but perform better in aesthetics uses	Epoxy resins are more likely to be used in high thermal applications but out of direct reach of the sun
Glass transition temperatures	
From 40°C to 110°C	From 40°C to 300°C
Costs and handling	
Not expensive and easy to handle	Highly expensive and present production difficulties
Toxicity	
Highly toxic	Less toxic

Epoxy resins are generally considered to have the best performance characteristics of all the resins used in the marine industry, being however more difficult to handle and highly expensive at the same time. These are derived from oxiranes and offer higher strengths from the previous being usually more rigid after cured appropriately. Despite their handling difficulties, these are considered less toxic with respect to their most uses, which may vary as pure resins for composite manufacturing or as additives for gelcoats and primers or painting undercoats. Their viscosity and transparency may differ from product to product depending on the aim of use, production techniques and temperature of storage.

1.1.3 Core materials

As previously anticipated, in order to save weight and perform particularly well when subject to bending loads, composite skins can be laminated to a low density core material. This technique is generally achieved in order to address axial and tangential stresses to different parts of the structures. The use of a sandwich, rather than an homogeneous material, aims in fact to concentrate axial loads to the skins and tangential ones to the core as compared below.
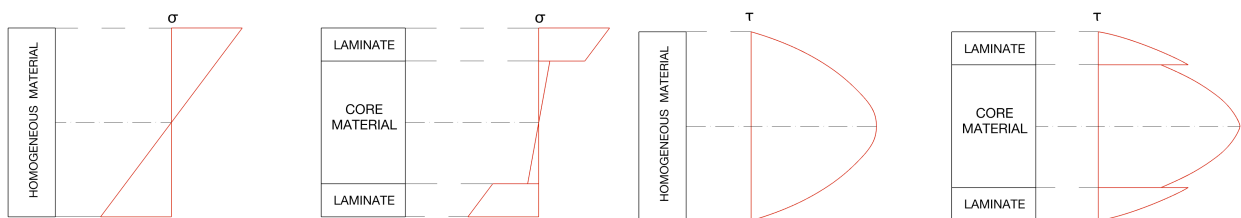
Figure 1.3: Axial (left) and Tangential (right) loads on non-homogeneous structures

Despite assuming a linear behavior of each material of the sandwich, the non-homogeneity of the structure results in a different stress distribution with respect to the homogeneous one of the steel beams. This behavior is defined by the different elastic modulus of the core material and the laminate (assumed with good approximation to be homogeneous).

Widely core material used in the sandwich fabrication may vary from different type of woods, foams or fiber structures.

Within the most used woods, the lightest one used for core materials is usually Balsa (around 150 kg/m^3) which has unidirectional strength due to its micro honeycomb structure. This last one is also reproduced on large scale from aluminum extrusions (between 30 and 90 kg/m^3) or Kevlar fibers (within 30 and 50 kg/m^3, also named as NOMEX). Its shape suits perfectly the core purposes exposed above, giving outstanding tangential reinforcement and extremely light structures.
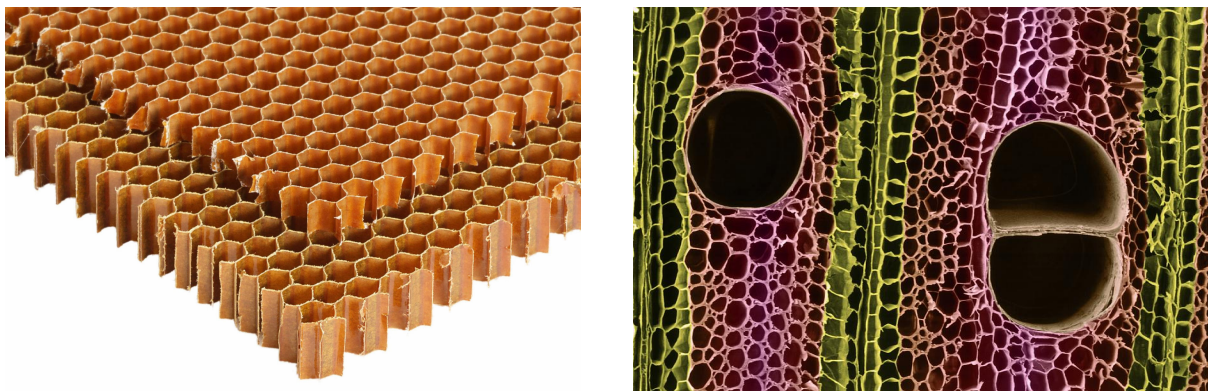
Figure 1.4: NOMEX (left) and Balsa wood (right at microscope) honeycomb structures for core materials

6

Within the possible foams for sandwich materials, the most used are PVC and Polymide, both varying their weight between 45 and 400 kg/m^3. The polyvinyl foam has great isotropic mechanical performances, low resin uptake and costs. It is definitely the most used foam in therms of sandwich core materials thanks to its ease-of-use and handling values. On the other hand Polymide foams are characterized by very high mechanical performances being at the same time very light but also expensive.
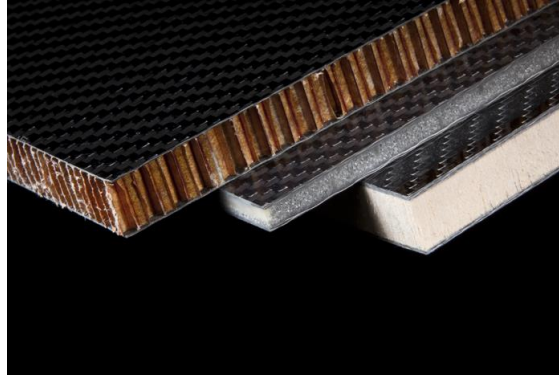
Figure 1.5: Carbon sandwich in NOMEX, PVC and Polymide (from left to right)

1.2   Fabrication techniques

Being effectively part of the structure, resins are to be considered for all intents and purposes characteristic elements with their own elastic modulus. It is fundamental in composite construction that whatever the scope of a structure is, the right ratio between resin and reinforcements is kept within the correct limits. A resin rich structure in fact may offer different less rigidity properties while a fiber rich one may lack of the sufficient matrix to be kept in place. In order to accomplish this task, different production techniques have been developed along the years which have lead to different results, not only from a structural point of view but also for production costs and timings.

Figure 1.6: Resin Infusion of a carbon fiber IMOCA hull in Persico shipyard

One of the most commonly used manufacturing techniques is the vacuum infusion. In this, dry materials (fibers) are laid into a mold and the vacuum is applied before resin is introduced. Once a complete vacuum and a drop test are achieved, the resin is literally sucked into the laminate via carefully placed tubing. This process not only allow to achieve the perfect resin to fiber ratio but, if performed correctly, avoids also imperfections into the structure caused by air traps. On the other hand however the resin infusion is a potentially difficult process which requires a high experience and appropriate vacuum pumps to be done

accurately. A ruined vacuum infusion may in fact results into irreparable laminates and expensive losses, especially on large scale pieces.

In order to avoid most issues with this last method, pre-preg materials have been developed in the last decade. This is a composite material made out of "pre-impregnated" (from which "pre-preg") fibers and a partially cured polymer matrixes. To allow easy handling in fact, these are just partially cured to their B-Stage where material requires cold storage in order to prevent the complete curing. This last one is done then when the fibers are laid out onto the mold and heated in large ovens under vacuum for several hours under controlled temperatures. As deductible, this technique is strongly limited to the shipyard availability of adequately sized ovens for the asset to be produced.

Figure 1.7: Pre-Preg carbon fiber sheets being laid out on a hull mold

Well stated the composite materials properties is now possible to introduce the new method. As will be subsequently shown, the developed procedure is specifically created to be modular and adaptive, such that at any time the user is able to opt for a material rather than another, while being at the same time customisable to the project for which is used. This being said, during the whole discussion will be made particular reference to the category of composite materials because the ones specifically suitable for the use of this method.

Chapter 2

A method overview

The extensive spreading of ships and pleasure crafts around the seas is progressively defining new project outlines, intensifying the process of study behind any possible aspect of a vessel, making them to perform always better and better, both from the owner and the builder points of view.

To provide an initial overview, merchant ships are continuously asked to be able to carry much more cargo than ever in order to better amortise fuel and operational costs with pollution benefits for the environment in which they sail. On the other hand, for example, a racing yacht may result much faster if structurally able to stand higher sailing loads being at the same time as light as possible. A large part of these aspects can be addressed to structural studies and, especially, to the corresponding scantling process. It is intuitive in fact that the cargo availability of a merchant ship and the performances of a racing vessel are strongly related to their structural capacities and spaces; this not only concerning the Naval architecture world, but also many different engineering fields of studies. Just thinking in fact to the highest building in the world or to a common F1 car is possible to identify many sharing structural problems which might be solved by using appropriately different solution and methods with the same basic procedures.

This being said, the designing time-frames might represent at the same time a high risk of costs increase for the shipyards. It is indeed in their own interests to profit as much as possible from any order, realizing an asset, as fast as possible, which performs closely to the design requirements without sacrificing the adopted building quality.

Stated now the expensively time and highly computational skills required for this process, the present discussion integrates perfectly into the theme, condescending the Naval Architecture needs, realizing a new powerful, easy-to-use and fast tool for weight-optimized structure scantlings.

2.1   Common scantling methods and issues

It is normally easy to perform any kind of stiffness verification beginning with given sets of structures and applied loads, to meticulously apply the needed building science theories and developed equations to check that the resulting internal stresses reside within the material mechanical limits. At the same time however it is not always possible to perform the above process backwards to identify the minimum dimensions a structure needs to have to stay within the material capacities given a first set of applied loads. This last procedure is generally speaking named *structural scantling*.

Is often possible anyways, for given sets of external loads and materials, to define the minimum *Section Modulus* a generic beam must have to resists without reaching its mechanical limit. The *Section Modulus* is a geometric property used in beams, reinforcement or any flexural members design to define their *load resistance* (improp.). The use of this modulus partially solves the problem of bounding the geometric required sizes of a given structure to their applied loads, but does not define numerically its dimensions, which in fact cannot be directly assessed. In other words, *hypothetically* infinite combinations of shapes

and sizes correspond to a single value of section modulus and for this reason is not possible to develop a single equation that creates an one to one joint between these three variables as the following:

$$W = f(DIM, SHAPE) \quad \& \quad DIM = f(W, SHAPE) \quad \& \quad SHAPE = f(W, DIM)$$

Stated this, it should be clear why the structural dimensions and shape evaluation of a generic system cannot rely singularly onto the section modulus knowledge, since at least one of these two elements is needed to evaluate the other.

$$(DIM, SHAPE) \implies W \quad but \quad W \nRightarrow (DIM, SHAPE)$$

This is why, in most of the cases, it is used to assume since the beginning a defined aspect of the beams cross sections, according to the past experience and production conveniences, for the dimensions to be effectively assessable. Once this has been fixed within reasonable limits, not being possible to explicate the function that provides the cross section dimensions ($DIM = f(W, SHAPE)$), an iterative process can be adopted to assign the given shape a set of feasible sizes and check that the obtained section modulus corresponds or overcomes the required one.

$$W_{obt} = f(DIM, SHAPE) \geq W_{req}$$

The entire process described above only sets the basis for the structural design, which in fact can be developed by looking forward to any possible optimization satisfying always the above inequality. Schematically speaking the former can be resumed in the following workflow:
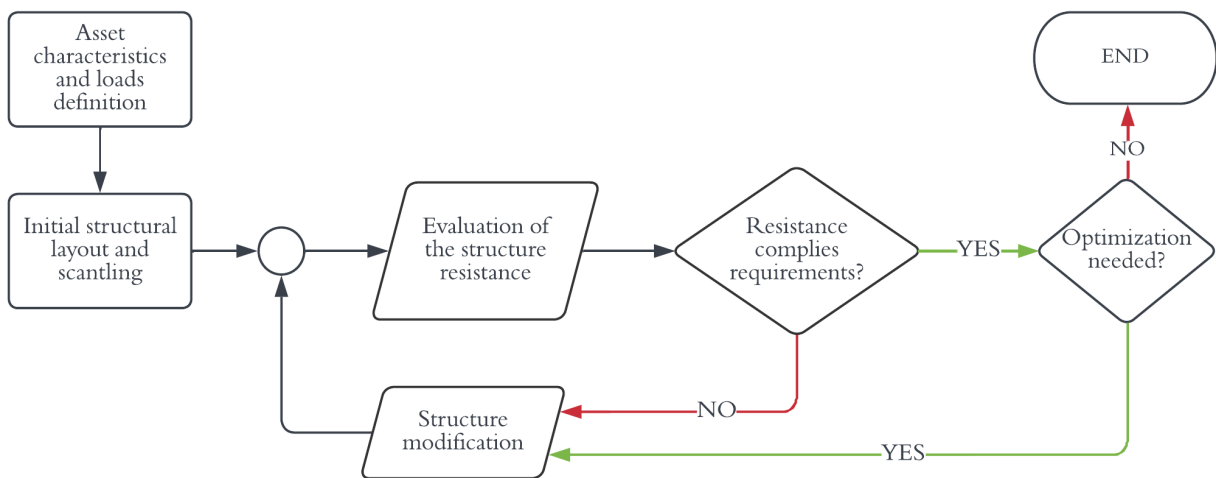
Figure 2.1: Widely used iterative structural sacantling method

Despite being always reliable and intuitive, the described method presents two well-known relevant concerns:

- Calculation timings;

- Solution convergence;

As previously said the designing process time-frame represents a relevant aspect of the asset production. In fact despite being convenient from the owner point of view, any optimization attempt takes designers a large time frame and, additionally, engages a large computational power (i.e. larger designing costs).

This being stated, large improvements have been made since the past times to speed up these processes by making use of different software which in any case can always be boosted in time.

Moreover the optimization process can be extended to as many as conveniently possible iterations with the awareness that the stopping criterion of the method relies exclusively to the human concern. It is a given fact indeed that, even if the the found solution is satisfactory to the designer opinion, this most likely does not correspond to the most optimal one both for weight and material costs.

The former aspects are not only relevant but represent design concerns the shipyards and owners are always in conflict with. The developed method introduces itself to obviate these issues by making use of advanced building science and mathematical theories integrated into powerful software available on the market.

2.2 Introduction to the method

The idea which solves partially these two concerns is represented by a powerful instrument able to create a structural frame sufficiently stiff and optimally light in a matter of a fraction of the needed time for the common method to perform a structural scantling. Even if the entire procedure will be subsequently more clear, the most convenient way to introduce it is to take a first look to the following scheme:
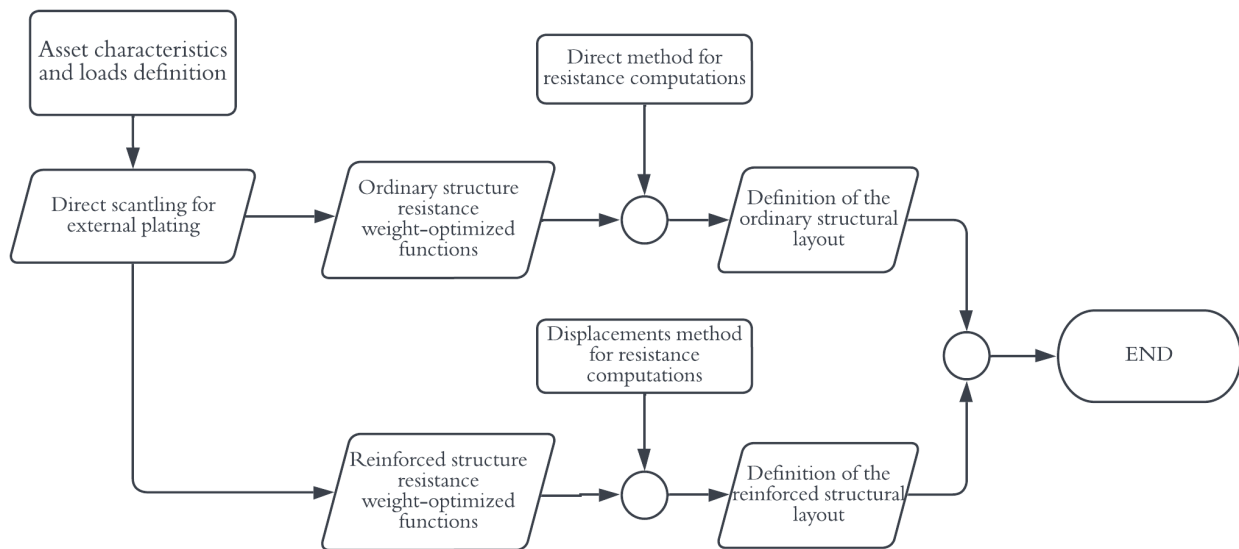
Figure 2.2: Alternative and optimized scantling method

The work-flow onto which the method is based, relies on the possibility to develop individually for each structural element, to which a single input requirement is given, weight-optimized numerical regressions. In this way the iterative procedure exposed previously is replaced with a linear extremely faster one and the method stops whenever a global minimum is identified in the domain.

2.2.1 Numerical regressions

In the Engineering world happens frequently to have a physical behavior pending onto multiple parameters which cannot be made explicit into an unique mathematical equation. In these cases, scientists frequently recur to different numerical methods able to describe, as closely as limited by the used models, the real performance of the phenomena.
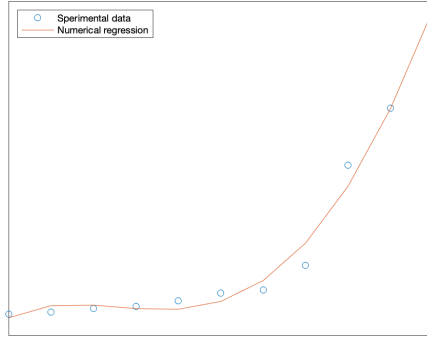
Figure 2.3: Simple example of a numerical regression

The use of a numerical regression for this project allows designers to obviate the manual iterative process regarding the structure scantling procedure beginning from a single W_{req}. By generating directly sets of beams of which the geometry and shape are defined *a priori*, is possible to evaluate the corresponding section modulus W to create a genuine relationship between the three variables.

Doing this, whenever is needed to know a set of dimensions and shapes of a given beam cross-section, is possible to use the up above functions by entering directly with the corresponding request W_{req}.

This being said it is worth to mention that, regardless these powerful numerical methods, it is not even close to possible to define equations that correlate any possible value of the section modulus W with a single cross section set of dimensions. This is due to the fact these two are not uniquely linked between each other but, in fact, for fixed values of W is possible to obtain almost infinite sets of reinforcement dimensions that, if applied to a cross-section, give the same section modulus value. In other therms, in order to evaluate the structure dimensions using this method the user should choose within infinite sets of numerical regressions, which makes it pretty much useless.

To solve this problem and identify the single beam to generate and associate to its corresponding W there is a very smart solution.

2.2.2 Weight optimization

A first option could be to fix a generic wide range of values for which each dimension may vary reducing significantly the possible combinations of these for the same W, especially if combined with the shipyard production constraints (even if the number of solution could not be inevitably reduced to one). Another much smarter one, instead, could be to introduce a criteria through which, within the possible combinations of these dimensions, only one is then sorted to be the most appropriate.

For the present method has been wisely chosen to go for the second option of the two above, where the idea is to sort out within the sets of available values the one that corresponds to a minimum weight of the single reinforcement and, so, of the complete structure. Despite in fact existing infinite possible combinations of the structure dimensions that give the same section modulus, to only one within these corresponds a minimum weight, or, at least, only one is then sorted.

$$W \iff ![(DIM, SHAPE)|\min(WEIGHT)]$$

In this way, not only the method results to be possible, but also correctly optimized, by making use of one of its major attribute (the weight minimization), as the criteria through which it is effectively applicable and reliable with no other additional approximation.

Well stated the method basis and fundamentals, it is now possible to proceed to deepen into the procedure itself and the software build up as it will be shown in the following chapters.

Chapter 3

Direct scantling Displacements method

The building science studies have brought along the years many different scantling criteria, differing between each others for ease of use, complexity, and realism. Advanced criteria may result in fact much closer to reality but at the same time more difficult to use and time consuming, while simplified methods are, in general quicker, but not optimized and overly conservative.

As far as the Ship construction is concerned, despite the large presence of conceptually different criteria, it is well known that any vessel must comply at any stage with the general requirements of the Classification Society they are registered to. These provide a specific documentation where any structural requirement is given specifically case by case, depending on the designed ship and different construction aspects.

Stated the fact for the developed software to be easily adjustable to any specific Classification rules, for the present project has been chosen to use the RINA 2007 Part B (for the Classification of Yachts) as main reference to critically test the effectiveness of the method. This regulation is specifically developed throughout a series of empirical formulations which can be used to directly compute the minimum requirements (section modulus) for every single element of a ship. Despite its simplicity and speed of use this method generates a structural frame which weight can be generally optimized whenever the solution is proofed to be resistant to buckling. For this reason we may distinguish two different scantling methods:

- Direct scantling method;

- Finite element displacement method;

3.1 Direct scantling method

The direct scantling method is namely a procedure through which a series of empirical or mathematically proofed equations is capable of identifying the minimum requirements of a single element of the structure. This method is generally used for evaluating the minimum thicknesses of external skins and plating or the required section modulus of secondary structural elements beginning with the applied local and global loads combined appropriately.

For the realization of a structural frame using this method, it is necessary to introduce the concept of structural hierarchy. This has the fundamental purpose of identifying the loads acting on each element that belongs to the grating and the constraint degree that characterizes it. Generally speaking, it is possible to think of skins and common beams as interlocking, since these have significantly lower levels of stiffness, to bottom, deck and bulkheads, or on girders and floors perpendicular to them. The choice of whether to classify transverse or longitudinal primary beams in the structural hierarchy is entirely arbitrary but needs to be made appropriately depending on the ship area under consideration.

Although the result may be much more conservative, when scantling elements belonging to higher orders in the hierarchy, we also want to take into account their constraint with the outer skins. Given a

generic load in fact, to exclude the presence of the internal tangential stresses caused by the bond between reinforced or secondary and the outer plates to which they are attached, could lead to misleading results. For this reason, we want to introduce the concept of the effective breadth, as that part of the external plating to which the structures are glued that permanently contributes to the strength of the individual beam.

On the basis of these considerations, a model is constructed in which each element supports the hierarchically lower and is supported at the same time by the above one. This assumption results to be usually more accurate as greater the difference in stiffness between elements of different order is and the more homogeneous are the considered loads. However, this does not necessarily imply that elements of a lower hierarchical order are less strong than others.

The figure shows an example of structural hierarchy. From this it can be seen how the spans of the secondary beams (blue arrows and loading area 2) are supported by reinforced floors (red arrows and load-bearing area 3), which are consequently interlocked to the corresponding girders (black arrows and load-bearing area 4) interrupted onto the transverse bulkheads. The loading area (1) corresponds to the skins one. This however, is irrelevant for the purpose of their thickness scantling, since this is exclusively dependent on the spacing of the beams supporting them.
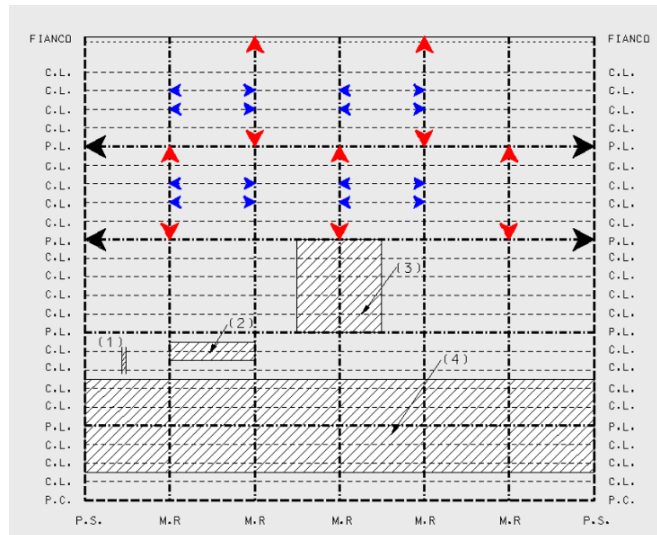
Figure 3.1: Example of structural hierarchy for longitudinal type structures

Once the beams span, loading area width and pressure reference points are defined is then possible to enter the data into the direct computation of the minimum requirements.

3.1.1 Pressure calculations

According to the place the structural element is positioned, the value of the scantling pressure is obtained by choosing the maximum values between the static (p_1) and dynamic (p_2) pressures (Chapter 1, Section 5 of the RINA regulations).

For planing and semi-planing yachts, the bottom, plating and stiffners design pressure is to be assumed as equal to the greatest of the values p_1 and p_2 defined as follows:

$$p_1 = \max\left(0.24 L^{0.5} \cdot \left(1 - \frac{h_0}{2T}\right) + 10 \cdot (h_0 + aL), 10D\right) \quad \& \quad p_2 = 15 \cdot (1 + a_v) \frac{\Delta}{L C_s} \cdot g \cdot F_L \cdot F_1 \cdot F_a \quad (3.1)$$

where h_0 is the vertical distance, in m, from the reference point to the full load waterline, a coefficient function of the longitudinal position of the same point, F_L, F_1 and F_a three coefficients given as a function

of the pressure point, the shape and inclination of the hull and the loading area supported by the stiffner respectively, while a_v the maximum vertical acceleration value at the considered cross-section.

For planing and semi-planing yachts, the side structure, plating and associated stiffners design pressure is to be assumed as equal to the value p_1 defined as:

$$p_1 = max\left(66.25 \cdot (a + 0.024) \cdot (0.15L - h_0), 10D\right) \tag{3.2}$$

For the side shells located forward of $0.3L$ from the forward perpendicular, the value p is to be not less than the value p_2 defined as follows:

$$p_2 = C_1\{k_v[0.6\sin(\gamma)\cos(90-\alpha)] + C_2 L^{0.5}\sin(90-\alpha)\}^2 \tag{3.3}$$

being C_1, C_2 two coefficients given as function of the loading surface and the longitudinal position of the considered element respectively, α the angle between the sides and the horizontal axis at the considered point, while γ the angle within the tangent at the draught waterline taken at the point of intersection of the transverse section of the element considered, with the above water- line and the longitudinal straight line crossing the above intersection (Figure 3.3).
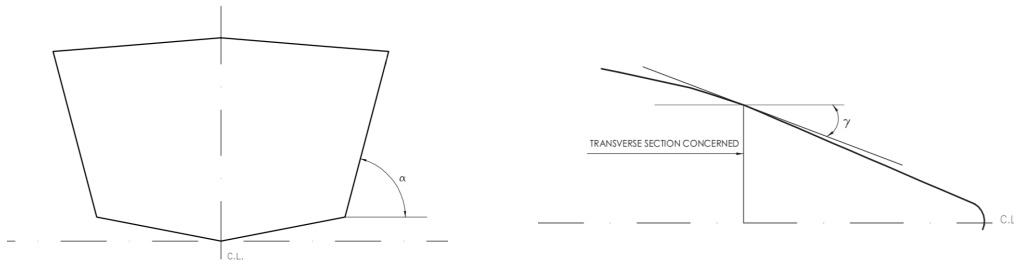
Figure 3.2: Definition of the α and γ angles from RINA

The RINA regulations also provide an useful table to evaluate the pressure heads h_0 for vessels decks as following:

	EXPOSED WEATHER AREA		SHELTERED AREA (also partially by deck-houses)
Deck	FWD 0,075 L from FWD PP	AFT 0,075 L from FWD PP	
	h_0	h_0	h_0
Deck below pdc	-	-	0,5
pdc	1,5	1,0	0,5
Decks over pdc	1,0	0,5	0,5

Figure 3.3: Definition of the deck pressure heads

to be assumed equally distributed with a mass density of $0.7 t/m^3$ and a consequent load per square metre of deck, in kN/m^2, equal to $6.9h_0$.

3.1.2 Skin thickness

Contrary to primary and secondary reinforcement elements, the minimum skin thickness is a direct function of the material mechanical characteristics, applied pressures and ordinary local spacing, and not of the type of structure (single or double bottom and longitudinal or transversal).

According to the RINA documentation (Page 165), the thickness of a single skin bottom plating is to be not less than the greatest between the values t_1 and t_2, in mm, obtained with the following equations:

$$t_1 = k_1 k_a k_{of} s p^{0.5} \qquad \& \qquad t_2 = 16 k_{of} s D^{0.5} \tag{3.4}$$

where k_1 and k_a are conservative factors defined according to specific tables, while k_{of} is a coefficient relative to the mechanical properties of the laminates (Page 171). If the bottom structure is build to be in sandwich material the RINA regulations provide an apposite equation for the outer and inner skin such as:

$$t_o = 0.50 \cdot (2.2 + 0.25L) \qquad \& \qquad t_i = 0.40 \cdot (2.2 + 0.25L) \tag{3.5}$$

The sides and deck laminate thickness are intended to be evaluated according to the provided equations for the sandwich structures, if a different type of structure is used then the appropriate formulae might be changed:

$$t_o = 0.45 \cdot (2.2 + 0.25L) \qquad \& \qquad t_i = 0.35 \cdot (2.2 + 0.25L) \tag{3.6}$$

Stated the use of the 3.6 equations for the side and weather deck shells, these outer plating will not be subject to optimization since their value fixed since the beginning, differently from the bottom one which can be further reviewed depending onto the longitudinals structure (equations 3.4).

3.1.3 Bottom longitudinals

With specific reference to a longitudinal type single bottom structure, according to the RINA specifications (Page 167), the minimum section modulus of longitudinal secondary beams is to be not less than the value Z, in cm^3, calculated with the following equation:

$$Z = k_1 s S^2 K_o p \tag{3.7}$$

being S the conventional span of the longitudinal stiffeners, in m, equal to the distance between floors.

3.1.4 Bottom floors and side frames

For the same type of structure as the last section, according to the RINA specifications (Page 167), the minimum section modulus of the floors at the center line of their span S is to be not less than the value Z_F, in cm^3, calculated with the following equation:

$$Z_F = k_1 b S^2 K_o p \tag{3.8}$$

where b corresponds to half the distance, in m, between the two floors adjacent to that concerned and S the conventional floor span equal to the distance, in m, between the two supporting members (sides, girders, keel with a dead rise edge grater than 12).

Similarly to this procedure is possible to evaluate the side reinforced frames by using the following:

$$Z_{SF} = k_1 K_{SF} s S^2 K_o p \tag{3.9}$$

where K_{SF} is a conservative coefficient defined according to the chosen hierarchy, s the spacing, in m, between the adjacent reinforced frames or, if absent, to the corresponding bulkhead, while S the conventional span, equal to the distance between the supporting members.

3.1.5 Bottom girders

Following for the same structure, according to the RINA specifications (Page 168), whenever a longitudinal reinforcement is supporting the floors according to the hierarchy, the minimum section modulus of central and side girders is to be not less than the values Z_{CG} and Z_{SG}, in cm^3, calculated with the following equations:

$$Z_{CG} = k_1 b_{CG} S^2 K_o p \qquad \& \qquad Z_{SG} = k_1 b_{SG} S^2 K_o p \qquad (3.10)$$

where b_{CG} and b_{SG} are half the distance, in m, between the two adjacent longitudinal girders or, if one is absent, to the corresponding side.

Following each equation described above is possible to obtain the minimum requirements for each element belonging to the modelled structure. From these is possible to proceed identifying the geometries of each cross section in order to satisfy the anticipated inequality $Z_{obt} \geq Z_{req}$ (Section 2.1). This passage will be deeply described in the next chapter according to the previously named numerical regressions.

3.2 Finite Element Displacement Method

The idea behind this such powerful approach is the approximation of the displacement field through a linear combination of *a priori* known polynomial functions and the creation of a new solutions class (kinetically congruent) in which the deformation of the solid body is described and a subclass in which these are also statically satisfied.

The method takes place in various steps as the following description shows.

3.2.1 Generation of the *Mesh*

Once the 3-dimensional model of the body frame is defined, the first step of the analysis is to subdivide the domain in a finite number of non overlapping elements (*Finite Elements*) joined together by *nodes*. This geometrical duplicate is also defined *Mesh* and it is characterized by a finite degrees of freedom given by:

$$n_{DoF,Mesh} = n_{nodes} * n_{DoF \forall node} \qquad (3.11)$$

3.2.2 Approximation of the displacement, deformation and stress field

At this second stage is possible to approximate the unknown displacement field for any of the finite elements e that compose the mesh throughout a linear combination of known functions whose coefficient correspond to the nodal displacements themselves. These functions are generally polynomial of the needed degree (i.e. depending on the element degrees of freedom) and are usually titled as *shape functions*:

$$\underline{u}^{e*}(\underline{x}^{e*}) \cong \underline{\hat{u}}^{e*}(\underline{x}^{e*}) = \underline{N}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \qquad (3.12)$$

where $\underline{N}^{e*}(\underline{x}^{e*})$ is the *shape matrix* which collects all the interpolation functions of the nodal displacements, while $\underline{a}^{e*}$ the coefficients vector of the up above linear combinations which correspond to the nodal displacements. The use of $e*$ notation is just to address the fact that all displacements $\underline{u}^{e*}$ are referred to the single element e in the itself body frame *.

The deformations follow immediately applying the congruence assumptions between displacements and the element deformations themselves:

$$\underline{\eta}^{e*}(\underline{x}^{e*}) \cong \hat{\underline{\eta}}^{e*}(\underline{x}^{e*}) = \partial \underline{\hat{u}}^{e*}(\underline{x}^{e*}) = \partial \underline{N}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \qquad (3.13)$$

where $\underline{B}^{e*}(\underline{x}^{e*}) = \partial \underline{N}^{e*}(\underline{x}^{e*})$ is usually defined as the *congruent deformation state approximation matrix* or *element strain-displacement matrix*. The 3.13 equations represent the fundamental mean between the approximate displacements and the following stresses, therefore they define the way any element of the mesh is supposed to deform under compelled loads.

The approximated stress field is defined applying the *constitutive equations* given by the structural mechanics theory and the deformation equations (3.13):

$$\underline{\sigma}^{e*}(\underline{x}^{e*}) \cong \hat{\underline{\sigma}}^{e*}(\underline{x}^{e*}) = \underline{C}^{e*}(\underline{x}^{e*}) \cdot \hat{\underline{\eta}}^{e*}(\underline{x}^{e*}) = \underline{C}^{e*} \cdot \underline{B}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \tag{3.14}$$

where $\underline{C}^{e*}(\underline{x}^{e*})$ is the *approximated stress field matrix*.

3.2.3 Definition of the static and deformation problem

At this stage all the ingredients which describe the static problem of the 3 dimensional body are defined. Once grouped together, if fulfilled, these equations represent the statically-satisfied solutions set:

$$\begin{cases} \hat{\underline{u}}^{e*}(\underline{x}^{e*}) = \underline{N}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \\ \hat{\underline{\eta}}^{e*}(\underline{x}^{e*}) = \underline{B}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \\ \hat{\underline{\sigma}}^{e*}(\underline{x}^{e*}) = \underline{C}^{e*} \cdot \underline{B}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \end{cases} \tag{3.15}$$

A kinetically-satisfied solution subset of the 3.15 clutch is then found by considering an infinitesimal variation of the nodal displacement $\delta \underline{a}^{e*}$ and applying it to the same equations.

The 3.15 problem can be developed alternatively applying the *Virtual Work Theorem* and obtaining a much easier-to-use formulation of the single finite element equilibrium. For the sake of brevity, the complete demonstration of what has just been said will not be reported, but only the final result which can be resumed in the following matrix equation:

$$\underline{f}^{e*} = \underline{K}^{e*} \cdot \underline{a}^{e*} \tag{3.16}$$

where $\underline{f}^{e*}$ represents the applied nodal forces vector, while $\underline{K}^{e*}$ the *stiffness matrix* defined individually for each element of the mesh. It is immediately clear that this matrix composed equation defines the kinetically congruent and statically well-balanced mathematical template solution for all finite elements, which represents the basis for the use of the *Finite Element Displacement Method*.

3.3 Displacement Method

The previous description details the procedure of what is commonly titled as *Finite Element Displacement Method*, which is nonetheless a general formulation of the *Finite Element Method* (or FEM) for structural analysis in therms of the nodal displacements.

It is notoriously frequent to further simplify the previous by making use of the *Displacement Method* whether the considered structure is made out of only one-dimensional beams either loaded uniformly along their length or exclusively at their ends. In this specific case there is no apparent need to subdivide each reinforcement into finite elements but it is convenient to consider them as the elements themselves of the mesh, being joined together by a couple of nodes placed at their ends. This is also a consequence of the fact that the shape functions of a linear elastic beam are actually exact displacement function.

These considerations allow to make use of exact *shape functions* without the need of any approximation of the displacement field. In other words the previous equation 3.12 becomes precisely:

$$\underline{u}^{e*}(\underline{x}^{e*}) = \underline{N}^{e*}(\underline{x}^{e*}) \cdot \underline{a}^{e*} \tag{3.17}$$

Resuming, the former *Finite Element Displacement Method* and the *Displacements Method* provide equal results whenever these are applied to particular structures as shown above, with the understanding that in the latter we have exact, known *a priori* and non-approximated *shape functions*.

3.4 Choice of the Method

Once it comes to choose whether of these three methods is the most correct to use, there is not a unique answer, considering that all three of these are able to perform a complete scantling procedure for a set structure.

This being said it should be clear at this stage how these may provide different results on the basis of their complexity of use. The two latter methods are to consider much more useful whenever weight optimization is a crucial variable of the project. Stated their much higher intricacy in fact, these allow to take into account the different interactions between different order structural element, which results to not be considered at all in the first one.

As will be shown in the following chapters, a considerably wise choice has been done distinguishing the used method depending on the structural area. As far as the outer plating and bottom longitudinal are concerned the student decided to adopt a *Direct Method* while refining the beams weight by making use of numerical optimized regressions. Whenever it comes instead to reinforced structure (Girders, Floors and Beams), a *Displacement Method* as been used together with appositely optimized numerical regressions, making advantage of the cited method for the *grid*-type structure without considering the contribution of the previous secondary one (plating and longitudinals).

This particular choice shall not be considered universally correct for any case of study, as the present project might be made extremely much more complex by coding the *Finite Element Method* and making use of the cited optimized numerical regressions for the whole structure.

Cleared this fact, is now possible to proceed describing the construction of the method itself.

Chapter 4

External plating and bottom longitudinals

Stated the use of the *Direct Method*, as seen in section 3.1 the bottom longitudinals and the outer skin thickness are strictly correlated within one to another, for this reason they will be optimized simultaneously. In particular, the present chapter will detail into any encountered problem along the optimization coding process and the effective solution adopted which allows the whole process to be performed beginning from the bottom structure itself.

4.1 Optimization methods and issues

In order to simplify the project with very good final results, the method used to dimension the minimum thickness and the bottom longitudinals is decided to be the direct one. Thanks to this has been possible to obtain the minimum thickness and section modulus requirement directly from equations 3.4 and 3.7.

In order to optimize at the best the two former elements is important to make the wisest choice by looking at the weight associated to each possible combination of the skin thickness and the longitudinals transversal area, both depending directly onto the secondary spacing s or, for each longitudinal strip within two adjacent girders, onto the number of longitudinals.

4.1.1 Cyclical optimization

It is clear how, from the same equations, a higher number of longitudinal elements would lead to thinner skins and smaller reinforcements. For this reason, a well-done optimization must be asked to investigate whether more or fewer elements solutions each strip would lead to more convenient options.

Independently from the number of variables, nonlinear optimization problems can be only solved whenever all of these are not dependent from each other.
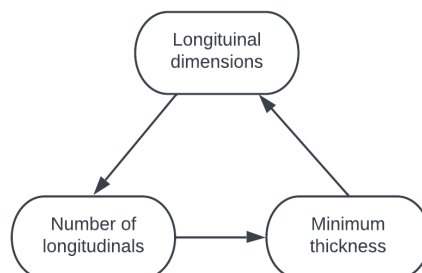
Figure 4.1: Cyclical optimization process

In this case instead the number of elements per strip is strictly related to their dimensions which according to the previous 3.7 equation, are defined by s.

This kind of optimization problems are generally named *cyclical optimization* and cannot be solved using canonical softwares functions.

4.1.2 Nonlinear optimization

The most convenient way to solve these problems is to perform a step by step optimization, for which at least one of the former is settled since the beginning of the said step. In this case, the fastest procedure is to fix the number of elements in each longitudinal strip and to perform each time the optimization process. In this way, at each step, the problem can be solved using nonlinear optimization methods.

Figure 4.2: Nonlinear optimization process

A nonlinear optimization (or nonlinear programming) is a mathematical problem of finding a vector x that minimizes the function f as:

$$\min_{x \in \mathfrak{R}^N} f(x)$$

subject to the constraints:

$$
\begin{aligned}
c(x) &\leq 0 \quad &\text{(nonlinear inequality constraint)} \\
c_{eq}(x) &= 0 \quad &\text{(nonlinear equality constraint)} \\
Ax &\leq b \quad &\text{(linear inequality constraint)} \\
A_{eq}x &= b_{eq} \quad &\text{(linear equality constraint)} \\
lb \leq x &\leq ub \quad &\text{(bound constraint)}
\end{aligned}
$$

where b and b_{eq} are vectors, A and A_{eq} are matrices, $c(x)$ and $c_{eq}(x)$ are x functions that return vectors, and $f(x)$ is a function that returns a scalar. Moreover $f(x)$, $c(x)$, and $c_{eq}(x)$ can be nonlinear functions.

4.2 Lagrange multipliers method

Given a nonlinear problem described by the sets of functions $g(x)$ and $h(x)$ as follows,

$$
\begin{aligned}
\min_{x \in \mathfrak{R}^N} &f(x) \\
g(x) &\leq 0 \\
h(x) &= 0
\end{aligned}
$$

The solution x_0 can be found by literally searching, for each array x, the one that satisfies the derivative test and results such to satisfy the given constraints. This mathematically equals to satisfy at a given solution x_0 the system of equations at next page.

$$\begin{cases} \nabla f(x_0) + \lambda g(x_0) = 0 \\ \nabla f(x_0) + \mu h(x_0) = 0 \end{cases}$$

The method of Lagrange multipliers is a strategy for finding the local maxima or minima of a function subject to equality and inequality constraints. The basic idea is to convert a constrained problem into a form such that the derivative test of an unconstrained problem can still be applied.

The relationship between the gradient of the function and the constraints rather naturally leads to a reformulation of the original problem, known as the Lagrangian function:

$$L(x, \lambda, \mu) = f(x) + \lambda g(x) + \mu h(x)$$

such that the derivative test results in a much easier way given by the following system of equations:

$$\nabla L(x, \lambda, \mu) = 0$$

In this way the original nonlinear constrained optimization problem becomes a non constrained one of the Lagrangian function $\nabla L(x, \lambda, \mu)$, which can be easily solved by using proofed methods as the *gradient descent* or the *Newton*'s one.

4.2.1 fmincon

These procedures can be done either manually or throughout a computing application as Excel or Matlab. Both of these have implemented solvers for nonlinear optimization problems and the given results depend strongly onto their computing power. For this Master thesis has been chosen to use the `fmincon` solver of Matlab but the results will be subsequently compared to an Excel users ones.

The `fmincon` function can be recalled into a Matlab workspace by using the following script:

```
1    [x,fval,exitflag,output] = ...
         fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options) %recalling fmincon ...
         solver for nonlinear optimization
```

where `fval` is a variable that returns the value of the objective function `fun` at the solution `x`, `exitflag` describes the exit condition of `fmincon`, and `output` provides a structure with information about the optimization process. The inputs `fun` and `x0` correspond to the function f to be minimized and to the initial point into which the solver starts. `nonloc` and `options` instead describe nonlinear constraints ($c(x) \leq 0$ & $c_{eq}(x) = 0$) and optimization options (such as the chosen algorithm, the step tolerance and the maximum function evaluations).

4.3 Bottom longitudinal strip optimization

In order to correctly optimize the bottom structure, identifying the optimization variables is a crucial initial point. Along this logical trend, the first step is to evaluate the lightest combination of:

- bottom outer skin thickness;

- longitudinals cross section dimensions; and

- number of longitudinals in each strip.

according to Figure 4.2.

The bottom outer skin thickness can be obtained directly using the 3.4 equations, but once obtained there are really no formal ways to optimize its value a part from computing it each time the main variables change. In this way each reinforced bottom panel between two girders and two floors will result to have a different thickness from another.

Stated the cyclical nature of the optimization, the number of longitudinals can't be directly used as an optimization variable, but will be chosen at each optimization process. For this reason the `fmincon` variables will be described to the code as a pure dimensions vector of the longitudinals cross-section only:

$$x = \begin{bmatrix} t_{web}, h_{web}, t_{flange}, b_{flange} \end{bmatrix}$$

corresponding to the thickness and height of the unified web and the thickness and width of the upper flange.

By making use of the equation 3.7 is possible to evaluate the minimum section modulus for a given number of longitudinals each strip (onto which depends their spacing s) and identify through `fmincon` the optimal combination of the x variables within defined nonlinear constraints which better optimizes the beams weight. In this way each reinforced panel between two girders and two floors results to have different cross section dimensions but same secondary spacing and effective breadth width of all those are on the same longitudinal strip.

From a coding point of view, the whole procedure above is made creating three singular loops which correspond at:

- selection of the longitudinal strip (`i` index);

- selection of a trial number of longitudinals (`l` index);

- selection of the transversal strip (`j` index);

At each round of the upper three cycles, the pressures and minimum thicknesses are evaluated recalling singularly the apposite made functions with RINA rules `p_min_RINA` and `t_min_RINA`:

```
1 %direct scantling of the outer skin bottom thickness
2 x_pdr_ord = x_bulkhead+dbb-(2*j-1)*S/2; %point of reference for pressure ...
      calclations [m]
3 [p1_ord,p2_ord,p_ord] = p_min_RINA(x_pdr_ord,displacement,s,...
4 S,beta_X,beta_LCG,h_0,C_S); %pressures calculations for RINA [MPa]
5 pressure_display = [pressure_display; n, l, x_pdr_ord, p1_ord, p2_ord, ...
      p_ord]; %displays pressures along n, l and pdr options
6 thickness_display = [thickness_display; n, l, x_pdr_ord, ...
      t_min_RINA(S,s,p_ord,p1_ord,p2_ord)]; %displays thickness along n, l and ...
      pdr options[mm]
```

`thickness_display` and `pressure_display` reports at a given number of longitudinals the corresponding applied pressures and minimum thicknesses of the reinforced panel. This last one is then used to identify a structural strip contributing structurally to each secondary reinforcement, which dimensions are defined by recalling the `fmincon` optimizer.

```
1 %defining requirements from RINA
2 Z_ord_min = Z_ord_RINA(S,s,p1_ord,p2_ord,p_ord); %minimum section modulus ...
      of each element [cm^3]
3
4 % Define the limits of the design variables [t_web h_web t_flange b_flange]
5 lb_ord = [0.2, 1.5, 0.2, 0.5]; %generic lower bounds for constructions ...
      limits [cm]
```

```
6  ub_ord = [4.0, b_e_ord, 4.0, 0.7*b_e_ord]; %generic measurements which ...
       might be subject to change for case to case [cm]
7
8  %defining optimization options
9  options = optimoptions('fmincon'); %solver selection
10 options.Algorithm = 'sqp'; %algorithm to use
11 options.Display = 'off'; %solution displaier switched off
12
13 x0_ord = lb_ord; %initial optimization point [cm]
14
15 [x_ord, area_ord, exitflag_ord, ¬] = fmincon(@ordinary_model, x0_ord, [], ...
       [], [], [], lb_ord, ub_ord, @ordinary_cons,options); %recalling fmincon ...
       for ordinary cross section optimization process
16 Z_min_tot = Z_min_tot_iter; %definition of the obtained section modulus ...
       after optimization [cm^3]
17
18 ord_optima_output = [ord_optima_output; n, l, x_pdr_ord, Z_ord_min, x_ord, ...
       area_ord, b_e_ord, t_e_ord, Z_min_tot];
19
20 %ord_optima_output = [number of floors, number of ord, x_pdr_ord, Z req, ...
       4Xdimensions, transversal area, b_e_ord, t_e_ord, Z obt]
```

The optimizer chooses each time the best cross section dimensions by beginning from their lower boundaries lb_ord. The used algorithm 'sqp' satisfies bounds at all iterations and can recover from infinite results. It is not a large-scale algorithm but is fast and uses a minimal quantity of memory. In the end ord_optima_output reports for each number of elements in a longitudinal strip the optimum dimensions of the ordinary structure cross section pending onto the number of floor, longitudinals and their reference point position.

Once the optimizer section is done, is possible to evaluate the total volume of longitudinals and outer skin for each strip given a number of beams. This done, is possible to select the solution which corresponds to a minimum overall weight.

The following code identifies the number of longitudinals each strip between two girders which brings the total weight to the minimum as said above.

```
1  vol_ord_per_strip(l,i) = sum(ord_optima_output((n+1)*(l-1)+1:(n+1)*(l-1)...
2  +(n+1),9))*l*S*100+sum(thickness_display((n+1)*(l-1)+1:(n+1)*(l-1)+(n+1),3))...
3  /10*S*100*dbg(i)*100; %evaluates volumes of longitudinals and outer skin ...
       for fixed number of longitudinals at each strip
4
5  [vol_min_per_strip(n,:),I] = min(vol_ord_per_strip,[],1,'linear');
6  [n_ord_per_strip(n,:),col] = ind2sub(size(vol_ord_per_strip),I); %selects ...
       each longitudinal strip the number of longitudinals for minimum volume
```

The above procedure will be repeated for a variable number of floors such that, given a structural reinforced frame each chunk, is always possible to find the solution that corresponds to a minimum weight of the ordinary stiffeners and outer skin.

From now, the next step is to select the optimum solution of the reinforced bottom girders and floors, side frames, deck beams and girders in order to get a minimum weight by making use of the *Displacement Method* together with appositely optimized numerical regressions, which will correspond this time to structural characteristic functions.

Chapter 5

Reinforced structural elements

The *Direct Method* described in Chapter 3 might be very simple to use and gives optimal results considering the minimal required effort, but does not really optimize the whole structure at its global minimum. This of course is due to the initial assumptions made to simplify and perform the said method, which doesn't consider the supporting interactions between each structural member. This being said, demanding a bit more effort, the *Displacement Method* can instead be used to scantle the reinforced members (Girders and Floors for the bottom, side Frames, Beams and deck Girders) at their minimum considering a variable number of transversal element at each chunk.

5.1 Application of the Displacements Method

There are several softwares available on the net which implement a 3-Dimensional workspace into which the user is able to design and represent the said structure and perform a structural analysis through which results is possible to optimize at the best the structure. Not being really accessible to everyone, these softwares will be all excluded for the present project, but a pure numerical approach will be followed. The associate Professor Marco Gaiotti at Univeristy of Studies of Genova has developed an easy to use Excel interface through which, given a structural reinforced frame geometry and the material characteristics, is able to identify the reinforcements cross sectional dimensions which suit at the best the required section modulus and realize the lightest overall chunk structure.

The way into which the *Displacement Method* has been particularly implemented into an Excel code won't be described, but will be following a simple description of its use.

5.1.1 Code inputs

In order to perform the structural reinforced frame analysis, the code is implemented to ask some simple basic inputs such as:

- distance between the two bulkheads;

- number of transversal elements;

- number and distances between longitudinal elements;

- outer skin thickness;

- main ship details; and

- material mechanical characteristics.

Through these the User is able to perform for each chunk and a given number of floors a complete optimization of the reinforced frame. In order to do this, two additional tables, one for longitudinal and the other for transversal elements, have been implemented into the code, which report the reinforcements cross section data (web thickness, net cross area, center of gravity net height, web height and flange cross area) obtained throughout a series of numerical regressions as function of the net cross section moment of inertia (without the effective breadth). Through these the section modulus are then evaluated for each element until the minimum structural requirements are satisfied and the structure results to be optimized.

At a first look, it might be spontaneous to question about the fact of numerically regress the cross sectional dimensions onto to moment of inertia instead of the section modulus directly. The reason why this approach has been followed relies in the numerical regression meaning itself. By using a set of dimensions regressed from an input variable, it is not strictly said that the numerical regressions obtained for each one of these behave in the same way. This is because of course a numerical regression is nonetheless an approximation of a given discrete trend, which affinity increases with the complexity of the regression itself.

For this reason the corresponding value of the section modulus for a given set of variables might not satisfy the minimum requirements even if the regressed one would do that. By using the single dimensions like this is then possible to rebuild automatically each iteration the cross section and evaluate the real modulus which most probably would differ from the regressed one. Doing this way not only any errors related to the numerical regressions approximation are solved, but also there is no need for further section verification since this is later built from scratch each time.

The former numerical regressions are made by generating a series of cross section, appositely optimized in their dimensions to fit at the best a given section modulus. The whole procedure is enveloped in a Matlab code which makes use of the functions `fmincon` and `polyfit` to evaluate the polynomial regressions onto the above data as function of the net cross section modulus of inertia.

5.1.2 Structural frame analysis

According to the procedure exposed previously in section 3.2, the first stage to apply the *Displacements Method* is to create a structural mesh through which the code is able to perform the numerical analysis. Since the considered domain is composed of the reinforced elements only, its subdivision into a finite number of elements is quickly done by considering each chunk of beam within two perpendicular ones as a separate element joined to the others by a couple of nodes, as previously cited within the application hypothesis of the present method. By doing this, each reinforcement results to be hinged at the bulkheads and sides, while free to move vertically at each inner joining node and to rotate along each transversal and longitudinal own axis. In reality these flexural and rotative contribution of each element creates a torsion stress onto the perpendicular ones onto which it is connected, but this can be generally neglected for the sake of simplicity. The stiffness matrix of equation 3.16 ($\underline{K}^{e*}$) is given immediately by using the degrees of freedom equations at each node of the structure.

In order to complete the 3.16 equation and explicit the displacements vector $\underline{a}^{e*}$, the next step is to evaluate the applied forces at each node. This is easily done by beginning with the applied pressures described previously according to the RINA regulations and identifying the loading areas associated to each reinforcement element and equally distributing the pressures along their length.

Once done the displacement vector is easily explicited by inversion of equation 3.16:

$$\underline{a}^{e*} = [\underline{K}^{e*}]^{-1} \cdot \underline{f}^{e*} \tag{5.1}$$

These are used now to evaluate the minimum requirements according to the building science theories as follows.

5.1.3 Minimum requirements

The use of the *Displacement Method* has been specifically used to identify the kinetically congruent and statically well-balanced solutions, for this reason each vector resulting from the previous 5.1 equation is automatically such to satisfy the dynamic beam equations. These relate the applied internal loads with the field of displacement associated to the considered beam and follow as:

$$M_{f,long} = \frac{6E \cdot J_{long}}{S_{long}^2}\epsilon_{vert} - \frac{2E \cdot J_{long}}{S_{long}}\gamma_{trans} \tag{5.2}$$

$$M_{f,trans} = \frac{6E \cdot J_{trans}}{S_{long}^2}\epsilon_{vert} - \frac{2E \cdot J_{trans}}{S_{trans}}\gamma_{long} + \frac{ql^2}{12} \tag{5.3}$$

where E represents the Young's module, J the own barycentric horizontal axis moment of inertia, S the beam length between two adjacent nodes while ϵ_{vert} and γ the vertical displacement and rotation around a longitudinal or transversal axis.

From these the minimum section modulus requirements for the longitudinal and transversal elements are obtained by the following:

$$W_{long} = \frac{M_{f,long}}{\sigma_{adm} - \sigma_{x1}} \tag{5.4}$$

$$W_{trans} = \frac{M_{f,trans}}{\sigma_{adm}} \tag{5.5}$$

where σ_{adm} represents the admissible tensile strength of the used material while σ_{x1} the global tension applied onto the whole structure due to global vertical moment.

5.1.4 Excel solver

Excel provides a useful instrument through which is possible to solve multivariate optimization problems as the one in exam. In this case the optimization variables correspond to the moment of inertia of the net cross section through which the numerical regressions of the beams dimensions are expressed (section 5.1.1).

The *Solver* is recalled by the apposite window in the Excel user upper interface *Data*. This asks for the aiming cell to minimize and the different constraints. In this case not many boundaries are necessary a part from the domain of the moment of inertia into which the regression has been made and the mandatory requirement that the available section modulus must result to be larger than the required ones, both for the longitudinal and transversal elements.

The solver than searches for the optimal solution and displays the resulting values of the moment of inertia and weight of the complete structural reinforced frame.

5.2 Optimized numerical regressions

In order to complete the exposed procedure, is necessary to input into the excel code the numerical regressions made for the reinforced structural elements. The code is expressively made to read forth degree polynomial regressions by directly their coefficients such as:

$$p(J) = p_1 J^4 + p_2 J^3 + p_3 J^2 + p_4 J + p_5$$

where J represents the net cross section moment of inertia, while $[p_1, p_2, p_3, p_4, p_5]$ the array that collects the regression coefficients required by the excel code.

In order to apply the method to both transversal and longitudinal reinforced elements of the bottom, side and deck structure, the first step to evaluate the numerical regressions required is to define the effective breadth contributing to the rigidity of the single elements. This is done by the following codes.

```
1  t_e_girder = t_e_bottom; %[cm] %effective breadth thicknesses
2  t_e_floor = t_e_bottom; %[cm]
3  t_e_side = sum([max([0.45*(2.2+0.25*L),4]) max([0.35*(2.2+0.25*L),3])])/10;
4  t_e_deck = t_e_side;
5  S_girder = dbb/(n+1); %[m] lenghth of the bottom girder
6  S_floor = B/(m+1); %[m] lenghth of the bottom floor
7  S_deck_girder = S_girder; %[m] lenghth of the side frame
8  S_deck_beam = B/(m_deck+1); %[m] lenghth of the deck girder
9  b_e_girder = min([0.2*S_girder;S_floor])*100; %[cm] effective breadth widths
10 b_e_floor = min([0.2*S_floor;S_girder])*100; %[cm]
11 b_e_side = min([0.2*D;S_girder])*100; %[cm]
12 b_e_deck_girder = min([0.2*S_deck_girder;S_deck_beam])*100; %[cm]
13 b_e_deck_beam = min([0.2*S_deck_beam;S_deck_girder])*100; %[cm]
```

Once done, the `fmincon` optimizer is recalled by defining its options and lower and upper boundaries

```
1  % Define the limits of the design variables
2  % [t_web h_web t_flange b_flange]
3  lb = [0.5, 0.5, 0.5, 0.5]; %generally speaking to avoid any kind of ...
       negative measurement [cm]
4  ub = [60.0, 200.0, 60.0, 200.0]; %generic measurements which shall be ...
       subject to change [cm]
5
6  %defining optimization options
7  options = optimoptions('fmincon');
8  options.Algorithm = 'sqp';
9  options.Display = 'off';
```

in order to optimize for each value of a fixed `W_range` array the cross section dimensions of each reinforcement:

```
1  W_range = (0:10:4000)'; %section modulus range into which perform the opt. ...
       [cm^3]
2  for k=1:size(W_range,1)
3
4      if k==1
5          x0_girder = lb;
6          x0_floor = lb;
7          x0_side = lb;
8          x0_deck_girder = lb;
9          x0_deck_beam = lb;
10     else
11         x0_girder = x_girder(k-1,:);
12         x0_floor = x_floor(k-1,:);
13         x0_side = x_side(k-1,:);
14         x0_deck_girder = x_deck_girder(k-1,:);
15         x0_deck_beam = x_deck_beam(k-1,:);
16     end
17
18     %optimization onto bottom girders
19     [x_girder(k,:), area_girder(k,1), exitflag_girder(k,1), ¬] =...
20         fmincon(@girder_model, x0_girder, [], [], [], [],lb, ub, ...
               @girder_cons,options);
```

```
21      Zg_girder (k,:) = z_G_net;
22      area_net_girder (k,:) = area_net;
23      J_y_net_girder (k,:) = J_y_net_iter;
24      W_min_tot_girder (k,:) = W_min_tot_iter;
25
26      %optimization onto bottom floors
27      [x_floor(k,:), area_floor(k,:), exitflag_floor(k,:), ¬] =...
28          fmincon(@floor_model, x0_floor, [], [], [], [],lb, ub, ...
                @floor_cons,options);
29      Zg_floor (k,:) = z_G_net;
30      area_net_floor (k,:) = area_net;
31      J_y_net_floor (k,:) = J_y_net_iter;
32      W_min_tot_floor (k,:) = W_min_tot_iter;
33
34      %optimization onto side reinforcements
35      [x_side(k,:), area_side(k,1), exitflag_side(k,1), ¬] =...
36          fmincon(@side_model, x0_side, [], [], [], [],lb, ub, ...
                @side_cons,options);
37      Zg_side (k,:) = z_G_net;
38      area_net_side (k,:) = area_net;
39      J_y_net_side (k,:) = J_y_net_iter;
40      W_min_tot_side (k,:) = W_min_tot_iter;
41
42      %optimization onto deck girders
43      [x_deck_girder(k,:), area_deck_girder(k,1), exitflag_deck_girder(k,1), ¬...
            ] =...
44          fmincon(@deck_girder_model, x0_deck_girder, [], [], [], [],lb, ub, ...
                @deck_girder_cons,options);
45      Zg_deck_girder (k,:) = z_G_net;
46      area_net_deck_girder (k,:) = area_net;
47      J_y_net_deck_girder (k,:) = J_y_net_iter;
48      W_min_tot_deck_girder (k,:) = W_min_tot_iter;
49
50      %optimization onto deck beams
51      [x_deck_beam(k,:), area_deck_beam(k,1), exitflag_deck_beam(k,1), ¬] =...
52          fmincon(@deck_beam_model, x0_deck_beam, [], [], [], [],lb, ub, ...
                @deck_beam_cons,options);
53      Zg_deck_beam (k,:) = z_G_net;
54      area_net_deck_beam (k,:) = area_net;
55      J_y_net_deck_beam (k,:) = J_y_net_iter;
56      W_min_tot_deck_beam (k,:) = W_min_tot_iter;
57
58  end
```

Differently from the previous longitudinals optimization, the reinforced elements one is done such that the optimization itself results into an as much as possible homogeneous regression. For this reason, despite being in any case completely free to dive into the variables domain, fmincon starts each optimization at the previous dimensions or, for the initial one, at the lower boundary values.

The results are then collected into arrays which define the corresponding values of the web thickness, net cross area, center of gravity net height, web height and flange cross area as function of the net cross section moment of inertia.

5.2.1 polyfit

From these is possible to evaluate the required numerical regressions by making use of the Matlab functions polyfit. This last one is a very powerful, mean squared error based method, to evaluate the $n+1$ coefficients of a polynomial function of nth degree which best fits (in a least-squares sense) the given data. The coding is done by recalling the following script.

```
1  p = polyfit(x,y,n)
```

where x and y represent the input arrays onto which make the regression $p(x)$ of the function $y(x)$, and n the required polynomial degree. In order to plot the obtained results the polynomial coefficients are used then together with the `polyval` function to evaluate the corresponding values $p(x)$ of the regressions. The following codes show the template used for the bottom floors only. This one has been consequently applied to all the remaining reinforcements.

```
1  p_A_floor = polyfit(J_y_net_floor,area_net_floor,4); %coefficients for A(J)
2  reg_A_floor = polyval(p_A_floor,J_y_net_floor);   %points of regression
3
4  p_Hw_floor = polyfit(J_y_net_floor,x_floor(:,2),4); %coefficients for Hw(J)
5  reg_Hw_floor = polyval(p_Hw_floor,J_y_net_floor);   %points of regression
6
7  p_tw_floor = polyfit(J_y_net_floor,x_floor(:,1),4); %coefficients for tw(J)
8  reg_tw_floor = polyval(p_tw_floor,J_y_net_floor);   %points of regression
9
10 p_Af_floor = polyfit(J_y_net_floor,x_floor(:,3).*x_floor(:,4),4); ...
        %coefficients for Af(J)
11 reg_Af_floor = polyval(p_Af_floor,J_y_net_floor);   %points of regression
12
13 p_Zg_floor = polyfit(J_y_net_floor,Zg_floor,4); %coefficients for Zg(J)
14 reg_Zg_floor = polyval(p_Zg_floor,J_y_net_floor);   %points of regression
15
16 PLOTTY
```

The values contained in the arrays p corresponding to each one of the five variables of the single reinforcement are then reported into the Excel to complete the structural scantling through the said solver.

The whole procedure is done for each number of floors chosen for the considered chunk in order to evaluate in the end which is the lightest solution. In addition, for each number of floor, this is then repeated for a variable number of deck girders affecting only the regressions for the deck reinforced elements.

This whole process puts the end to the structural optimizer exposition. The following chapters will show a case of study used as a test to check the functionality of the method.

Chapter 6

Method case of study

In order to check if the developed method is effectively functional and to look for any further improvements, a simple case of study has been chosen to perform the structural weights optimization.

The results are then compared to the ones obtained with another Excel-only-based numerical optimizer developed by the dear colleagues Eng. Edoardo Sommovigo and Eng. Andrea Munafo.

6.1 Vessel details

The examied vessel corresponds to Edoardo Sommovigo's Bachelor thesis, a performance yacht made out of fiberglass composite, single skin and sandwich depending onto the considered structure. Some of the basic details and a vessel general plan are reported following.

Table 6.1: Basic vessel details

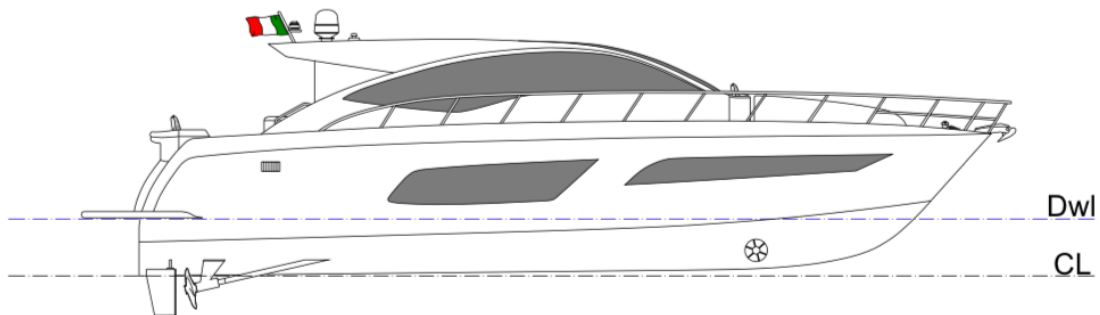|	
Length overall $[m]$	16.34
Breadth $[m]$	4.22
Moulded depth $[m]$	3.15
Design draft $[m]$	1.20
Displacement $[t]$	30
Service speed $[knots]$	20

Figure 6.1: Vessel general plan

6.2   Initial assumptions

In order for the project to be comparable with the other method, some simplifying assumptions were required. These actually do not constraint severely the optimization process but only exclude a couple of ulterior cases to be considered. It is important to highlight how, despite these assumptions, the user will be absolutely not limited on the use of the method but can adopt it to its own needs.

6.2.1   Building materials and global loads

To compare the structures obtained with the two methods, it is supposed to build the whole vessel in fiberglass, with foam core for the sandwich skins and reinforcements. The single skin and sandwich solutions have been subsequently combined depending onto the area (bottom, sides and weather deck) in order to see whether solution was more convenient.

This being said, any sandwich structure has been assumed to be composed by the only laminate, at least from a stiffness stand point of view, stated the very low Young's module of the foam core not really contributing to the rigidity of the structure itself.

In the end the assumed building material has resulted to have the following characteristics:

Table 6.2: Building material characteristics

Density $[t/m^3]$	ρ	1.7
Admissible tensile strength $[MPa]$	σ_{adm}	123.48
Young's Module $[MPa]$	E	$\sim 10^4$
Percentage of fibers in the laminate	G_c	0.4
Global longitudinal stress $[Mpa]$	σ_{x1}	0

The percentage of fibers in the laminate has been assumed equal to an ideal mean value stated the fact that, depending onto the fabrication technique adopted (section 1.2), a shipyard is generally not really able to choose between one or another but to use the one available for the construction.

The global longitudinal stress σ_{x1} has been further assumed to be equal to zero stated the reduced length of the considered vessel according to the RINA regulations.

6.2.2   Maximum number of structural elements

Any structural reinforcement onboard the vessel has been made to vary in number and dimensions within reasonable limits. This procedure ensures always to analyse any possible different structural layout, making the optimizer totally reliable at any time without compromising its functionality.

Stated the very closed-up girders distribution, the number of longitudinal elements within a couple of these has been made to vary within one and four, considering at each attempt the required space to not overlap into one another.

As far as the floors are concerned, a reasonable maximum number for each chunk has been established to five, even if, considering the reduced bulkheads distances, surely would have been possible to break the attempts at two or three in order to further speed up the scantling procedure. This fact will be much more clear in the following results chapters.

In order to complete a full optimization, different solutions of increasing number of deck girders have been analyzed to see whether it would have been possible to save some weight from the deck structures too. This consideration is further justified if, according to the RINA regulations, these are kept to be continuous along the chunks of the vessel as done with the bottom girders and floors, frames and beams for the same chunk.

Despite not being any apparent reason to choose such large ranges for the reinforcements quantities, the procedure is justified for this single case of study, to understand the effective power of the developed optimizer. As said previously in fact, the results will show how the more convenient solutions, from a weight point of view at least, will be always made out of much fewer structural elements, highlighting, as expected, how neither a lot or a few reinforcements correspond to the lightest combination.

For any future projects, the user is always asked to insert a maximum quantity of each element to analyse. The code will subsequently check automatically whether more reinforcements may result in lighter solutions or not.

6.2.3 Structural assumptions

The main structural assumptions concern the bulkheads and the bottom girders.

The firsts haven't been analyzed since their scantling process may be done according to the direct method using RINA regulations, moreover their position has been chosen to be the same of the original Sommovigo's project without considering their thicknesses. If a radical *Finite Element Method* would have been adopted surely these would have been subject to a deeper optimization by taking into account the reinforcements interactions.

The number and the position of the bottom girders have been fixed as customary on smaller vessels since the beginning. These in fact have been located precisely to support the other structural components, the main engine and respect the geometry of the inner spaces without compromising at all the yacht functionality.

The following tables report the position of the bulkheads from the aft perpendicular and the girders from the central base line.

Table 6.3: Chunk positioning from aft perpendicular

Selected chunk	Aft bulkhead $[m]$	Fore bulkhead $[m]$	Chunk length $[m]$
CH 1	0.000	5.272	5.272
CH 2	5.272	9.076	3.804
CH 3	9.076	11.598	2.522
CH 4	11.598	15.390	3.792

Table 6.4: Girders positioning from central base line

Selected chunk	Left girder $[m]$	LC girder $[m]$	RC girder $[m]$	Right girder $[m]$
CH 1,2,3	-1.45	-0.45	0.45	1.45
CH 4	/	-0.45	0.45	/

6.3 Optimization boundaries and constraints

As seen previously, the use of the Matlab function `fmincon` allows to chose both variables boundaries and optimization constraints. These should be always subject to adjustment at the user concern for the project under exam in order to provide correct results for the numerical regressions and the shipyard construction availability. The user is in fact always free to chose quickly into the workspace any further constraint necessary given by the project, the shipyard or the owner himself.

For the present project, it has been chosen to select some generic boundaries and constraints in order to see whether the code was able to find correct and optimal solutions even under limited ranges. Following a brief description of the adopted boundaries and nonlinear constraints.

Table 6.5: Longitudinals dimensions boundaries

Longitudinals variables	Lower bound	Upper bound
Web thickness $[cm]$	0.2	4.0
Web height $[cm]$	1.5	b_e
Flange thickness $[cm]$	0.2	4.0
Flange width $[cm]$	0.5	$0.7b_e$

Table 6.6: Primary reinforcements dimensions boundaries

Reinforcements variables	Lower bound	Upper bound
Web thickness $[cm]$	0.5	60.0
Web height $[cm]$	0.5	200.0
Flange thickness $[cm]$	0.5	60.0
Flange width $[cm]$	0.5	200.0

where b_e defines the effective breadth width.

Table 6.7: Structural cross section nonlinear constraints

$$t_e \leq t_{flange}$$
$$b_{flange} \leq h_{web}$$
$$5t_{web} \leq b_{flange}$$
$$0.5t_{web} \leq t_{flange}$$
$$0.5t_{web} \leq t_{flange}$$

6.4 Results

6.4.1 Secondary order structure

The Matlab source has been coded appositely to provide easy to manage charts each reinforced panel of a vessel chunk, to group up the corresponding longitudinals dimensions and outer skin thicknesses optimized to the requirements from a varying number of transversal floors. For the sake of brevity the whole charts won't be here reported because not very useful at all. The reader is asked now to envision a simple chart where by varying the number of floors, each reinforced panel, corresponding to a row of this, is is linked to by a number of longitudinals, a set of cross section sizes and a skin thickness which minimizes the full longitudinal strip weight.

By doing this way, whenever all primary reinforcements layouts will be analyzed, the corresponding solutions weight of equal number of floors between primary and secondary order structures will be summed up together and will be possible to identify the lightest combination of these.

6.4.2 Primary order Structure- Numerical regressions

In order to let Excel analyse the different structural reinforcement frames, the Solver needs the cross-sectional optimized numerical regressions to be entered throughout their coefficients.

These not only show how the optimizer has worked to find the best combination of the four cross-sectional size variables, but also give a preliminary idea on the numerical approximation made by `polyfit` to fit the optimized solutions.

Since the results are given for each chunk, varying the number of floors and deck girders, the following figures will show briefly the only chosen configuration which corresponds to a minimum overall weight for the first chunk only. These are in fact to be considered ideally equivalent for each chunk and number of reinforcements, because strictly related to the nonlinear constraints instead that to the surrounding layout.

Differently from the longitudinals optimization, the reinforcements numerical regressions report, as previously anticipated, the net cross-sectional area, the web height and thickness, flange area and centre of gravity height as functions of their moment of inertia.

6.4.3 Net cross-section area regressions
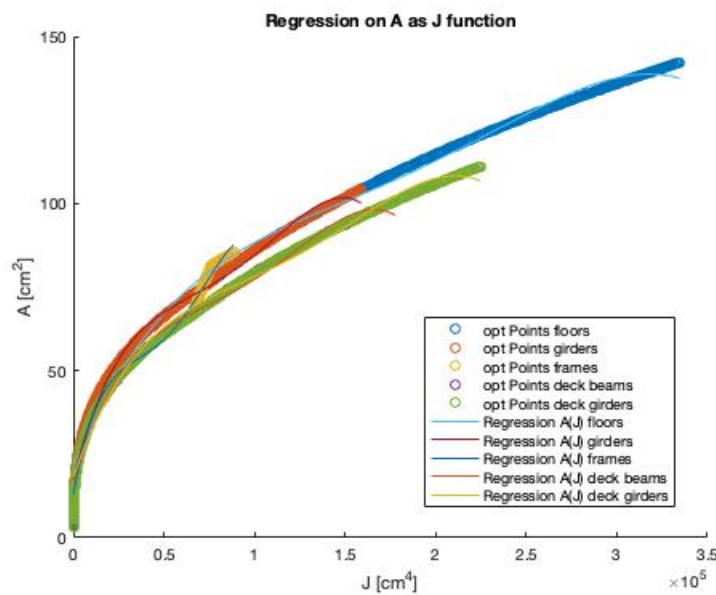
Figure 6.2: Net cross-section area regressions

Some small cases might recall more attention showing different trends to the others. This particular behavior is generally related to optimization nonlinear constraints. Given that Matlab always tries to push a subset of variables to their limit, as soon as a constraint takes over the trend tend to diverge from the other points. Despite this, the behavior shouldn't worry too much since there aren't any negative aspects for these being extremely optimized instead of keeping a continuous trend.

6.4.4 Web height regressions
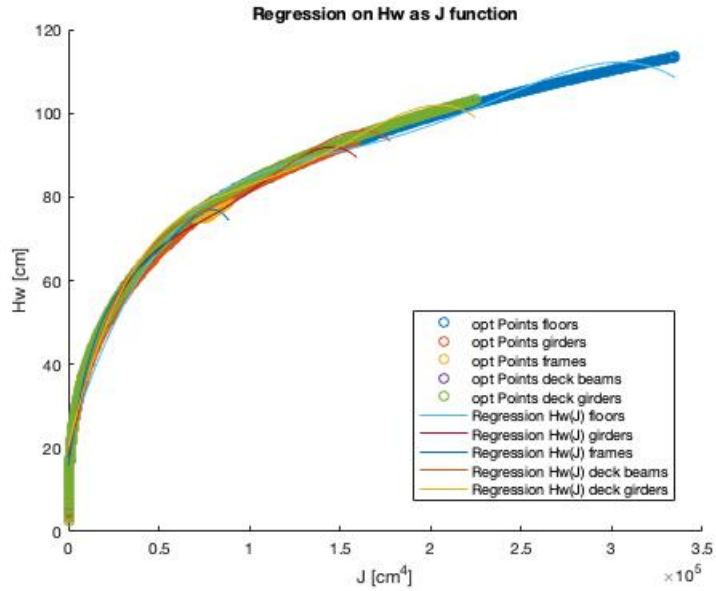
Figure 6.3: Web height regressions

6.4.5 Flange area regressions
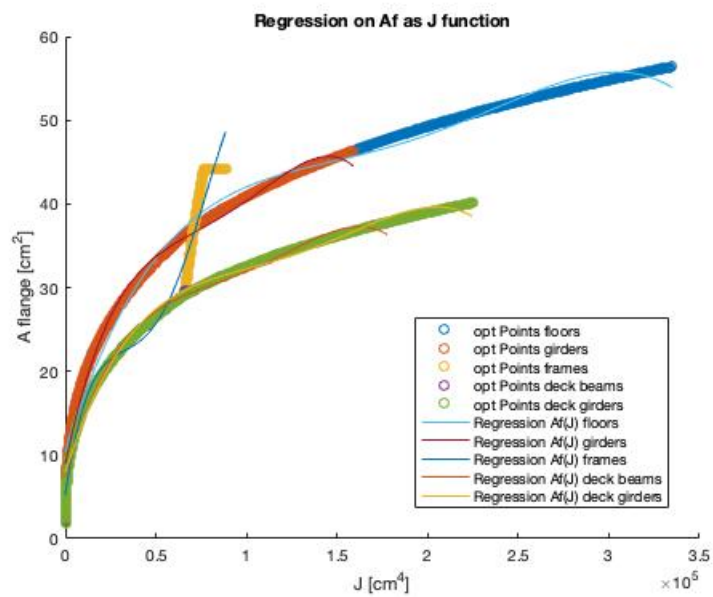
Figure 6.4: Flange area regressions

6.4.6 Web thickness regressions
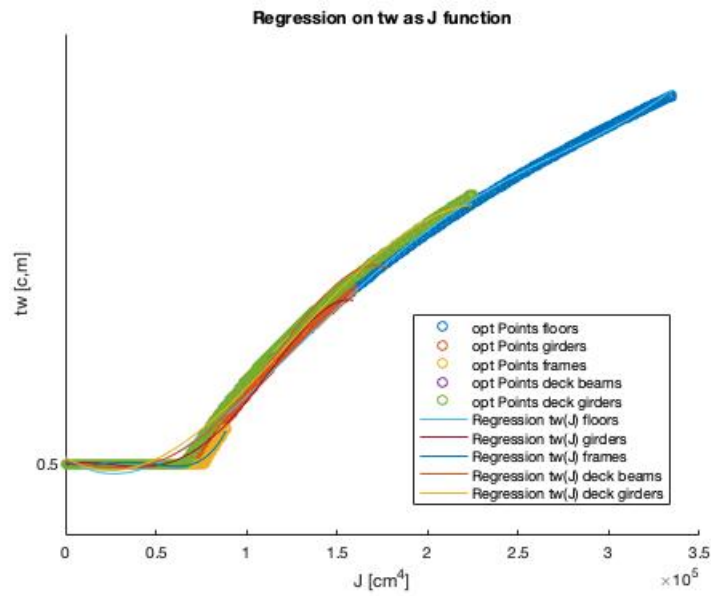
Figure 6.5: Web thickness regressions

Other examples as the above one might recall the reader attention by presenting non uniform trends. In this particular case, the web thickness lower bound has been set to 0.5 cm according to 6.6 table as imposed by RINA instructions, for this reason the optimization had to deal with other variables to fit appropriately the given section modulus.

6.4.7 Centre of gravity height regressions
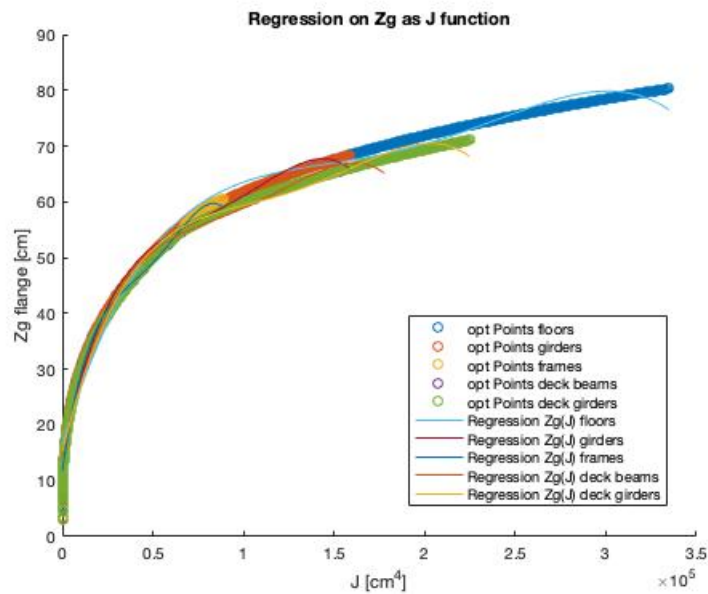
Figure 6.6: Centre of gravity height regressions

Despite the very large domain into which the optimizer is able to find the lightest combination of the above variables, the obtained trends show actually how the software works very well to find always the most correct solution to the optimization problem without highlighting weird and spread data trends.

This being said, despite a couple of small localised divergences, the numerical regressions seem to work fairly well even if presenting slightly different values from the optimized ones. Any of these errors will be subsequently fixed by the Excel Solver.

These regressions are so placed into the Excel code and used by the Solver to find the optimized reinforcements for each chunk varying the number of transversal elements.

6.5   Results - Final comparison

The Excel runs each time a different numerical regression associated to a particular structural layout.

The following tables show the results obtained with the only Excel based method and the present Matlab based one.

6.5.1   Bottom structure

Table 6.8: Bottom structure weights from the Matlab-based method - First chunk

Nr. of Floors	Girders $[kg]$	Floors $[kg]$	Longitudinals and skin $[kg]$	Total $[kg]$
1	200	22	289	511
2	136	43	288	467
3	142	63	287	492
4	131	83	285	499
5	130	104	279	513

Table 6.9: Bottom structure weights from the Excel-based method - First chunk

Nr. of Floors	Girders $[kg]$	Floors $[kg]$	Longitudinals and skin $[kg]$	Total $[kg]$
1	226	31	335	592
2	192	43	337	572
3	202	56	335	593
4	191	66	335	592
5	193	77	336	606

Table 6.10: Bottom structure weights from the Matlab-based method - Second chunk

Nr. of Floors	Girders $[kg]$	Floors $[kg]$	Longitudinals and skin $[kg]$	Total $[kg]$
1	83	21	206	310
2	68	42	205	315
3	71	62	203	336
4	68	83	203	354
5	68	104	202	374

Table 6.11: Bottom structure weights from the Excel-based method - Second chunk

Nr. of Floors	Girders [kg]	Floors [kg]	Longitudinals and skin [kg]	Total [kg]
1	119	27	263	409
2	107	40	256	403
3	114	53	257	424
4	108	65	259	432
5	109	76	261	446

Table 6.12: Bottom structure weights from the Matlab-based method - Third chunk

Nr. of Floors	Girders [kg]	Floors [kg]	Longitudinals and skin [kg]	Total [kg]
1	35	21	70	126
2	37	42	64	143
3	38	62	62	162
4	39	83	58	180
5	40	104	56	200

Table 6.13: Bottom structure weights from the Excel-based method - Third chunk

Nr. of Floors	Girders [kg]	Floors [kg]	Longitudinals and skin [kg]	Total [kg]
1	45	22	185	252
2	41	34	187	262
3	44	46	189	279
4	45	55	191	291
5	45	70	191	306

Table 6.14: Bottom structure weights from the Matlab-based method - Fourth chunk

Nr. of Floors	Girders [kg]	Floors [kg]	Longitudinals and skin [kg]	Total [kg]
1	41	15	70	126
2	26	33	65	124
3	37	41	65	143
4	45	42	58	145
5	48	47	56	151

Table 6.15: Bottom structure weights from the Excel-based method - Fourth chunk

Nr. of Floors	Girders [kg]	Floors [kg]	Longitudinals and skin [kg]	Total [kg]
1	18	27	207	252
2	18	51	207	276
3	18	67	210	295
4	18	82	213	313
5	18	92	215	325

From a first comparison of the bottom structure it is clear how between the two scantling methods, the Matlab based one is able, in any possible configuration, to find lighter solutions than the second one.

This behavior is particularly due to the large differences in the girders and longitudinals weights. This highlights how, not only Matlab numerical regressions are much closer to the developed data, but also the higher computing power is able to optimize the structures cross section structure much better than Excel.

By looking at the tables it results in fact that Matlab is making out a mean weight saving in the bottom structure of almost the 35% than Excel, with a minimum save of 14% and a maximum one of 55% in a couple of configurations.

6.5.2 Side structure

Table 6.16: Side structure weights from the Matlab-based method - First chunk

Nr. of Floors	Frames $[kg]$	Skin $[kg]$	Total (P+S) $[kg]$
1	3	143	292
2	5	143	296
3	8	143	302
4	11	143	308
5	14	143	314

Table 6.17: Side structure weights from the Excel-based method - First chunk

Nr. of Floors	Frames $[kg]$	Skin $[kg]$	Total (P+S) $[kg]$
1	4	143	294
2	7	143	300
3	9	143	304
4	12	143	310
5	15	143	316

Table 6.18: Side structure weights from the Matlab-based method - Second chunk

Nr. of Floors	Frames $[kg]$	Skin $[kg]$	Total (P+S) $[kg]$
1	3	103	212
2	5	103	216
3	8	103	222
4	11	103	228
5	14	103	234

Table 6.19: Side structure weights from the Excel-based method - Second chunk

Nr. of Floors	Frames $[kg]$	Skin $[kg]$	Total (P+S) $[kg]$
1	3	103	212
2	6	103	218
3	9	103	224
4	12	103	230
5	15	103	236

Table 6.20: Side structure weights from the Matlab-based method - Third chunk

Nr. of Floors	Frames [kg]	Skin [kg]	Total (P+S) [kg]
1	3	66	138
2	5	66	142
3	8	66	148
4	11	66	154
5	14	66	160

Table 6.21: Side structure weights from the Excel-based method - Third chunk

Nr. of Floors	Frames [kg]	Skin [kg]	Total (P+S) [kg]
1	3	66	138
2	6	66	144
3	8	66	148
4	11	66	154
5	14	66	160

Table 6.22: Side structure weights from the Matlab-based method - Fourth chunk

Nr. of Floors	Frames [kg]	Skin [kg]	Total (P+S) [kg]
1	3	99	204
2	5	99	208
3	8	99	214
4	11	99	220
5	14	99	226

Table 6.23: Side structure weights from the Excel-based method - Fourth chunk

Nr. of Floors	Frames [kg]	Skin [kg]	Total (P+S) [kg]
1	3	99	204
2	6	99	210
3	8	99	214
4	11	99	220
5	14	99	226

All the results are still reported as function of the number of floors since a prerogative of the RINA regulations is that the reinforced structural elements must be continuous along the ship, so that the bottom, sides and weather deck belonging to the same chunk contain an equal number of transversal elements as well as the number of reinforced girders are equal along the length of the ship unless its width might become too tight (as for the fourth chunk in table 6.4).

From the tables it results clear how between the Matlab based and the Excel based method there isn't such a large advantage (in a least-total-weight sense). This result is definitely due to the fact that along the sides structures the reinforced frames represent the only element that can be optimized, recalling that the sandwich outer skins are defined directly according to the classifications by making use of the *Direct Method* and cannot be optimized as much as the bottom single skin.

Moreover, the very small side frames do not leave room for intensive optimisations since, independently from the used model and optimization tools, all cross-sections numerical regressions tend to converge to the same minimum structural dimensions. It should be obvious in fact why the largest save between the two methods in any possible side structural layout corresponds at least to the 1%.

6.5.3   Weather deck structure

The procedure followed for the weather deck structure has been slightly changed. In fact recalling the possibility to change the number of longitudinal reinforced deck elements, stated that these must be in equal number along the ship, all the possible configurations have been analyzed by the Excel optimizer and have been implemented together with the other results (bottom and sides) in order to evaluate which was the most convenient solution of number of deck girders and bottom floors.

In the end it was clear that for this small vessel the lightest of the solutions was actually corresponding to the single deck girder one, despite the weights percentage differences were roughly of the 2%.

The following results show in this way the solutions obtained for the single deck girder option.

Table 6.24: Deck structure weights from the Matlab-based method - First chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	14	13	226	253
2	16	26	226	268
3	18	40	226	284
4	18	53	226	297
5	19	66	226	311

Table 6.25: Deck structure weights from the Excel-based method - First chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	6	6	226	238
2	6	11	226	243
3	6	17	226	249
4	6	22	226	254
5	6	28	226	260

Table 6.26: Deck structure weights from the Matlab-based method - Second chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	12	13	163	188
2	13	26	163	202
3	13	40	163	216
4	14	53	163	230
5	14	66	163	243

Table 6.27: Deck structure weights from the Excel-based method - Second chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	5	5	163	173
2	5	11	163	179
3	5	16	163	184
4	5	22	163	190
5	5	27	163	195

Table 6.28: Deck structure weights from the Matlab-based method - Third chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	9	13	108	130
2	9	26	108	143
3	9	40	108	157
4	9	53	108	170
5	10	66	108	184

Table 6.29: Deck structure weights from the Excel-based method - Third chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	3	5	108	116
2	3	11	108	122
3	3	16	108	127
4	3	22	108	133
5	3	27	108	138

Table 6.30: Deck structure weights from the Matlab-based method - Fourth chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	12	13	163	188
2	13	26	163	202
3	13	40	163	216
4	13	53	163	229
5	14	66	163	243

Table 6.31: Deck structure weights from the Excel-based method - Fourth chunk

Nr. of Floors	Girders $[kg]$	Beams $[kg]$	Skin $[kg]$	Total $[kg]$
1	5	5	163	173
2	5	11	163	179
3	5	16	163	184
4	5	21	163	189
5	5	27	163	195

The different trend which shows up how the excell based method is actually providing lighter solutions to the Matlab one is quickly justified by considering the two different minimum moments of inertia constrained to the Excell solver, which have resulted to be higher in this scantling attempt.

6.6 Total weight results and comparison

By analyzing singularly the different options of number of deck girders and bottom floors, the lightest combination of these two variables has resulted to be the one corresponding to a single deck girder (as anticipated) and a single floor in any chunk with the exception of the first one which resulted to be conveniently built with two smaller ones.

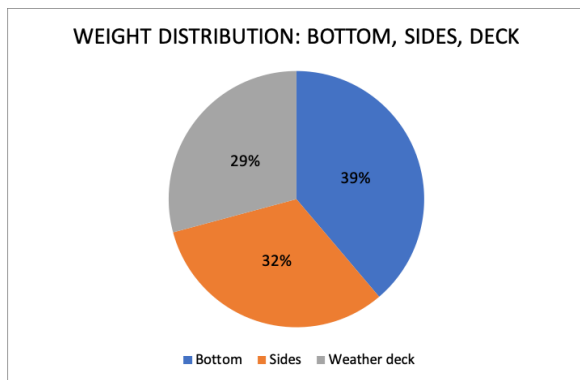The following charts resume the obtained results.

Table 6.32: Weight distributions (1 out of 3)
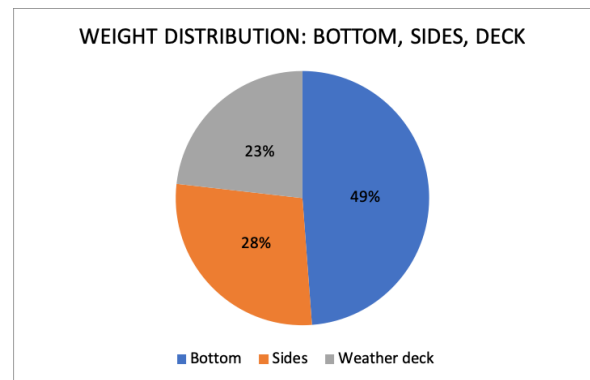
(a) Matlab-based results

Element	Weight $[Kg]$	Percentage [%]
Bottom	1029	39
Sides	850	32
Deck	774	29
TOT	2653	

(b) Excel-based results

Element	Weight $[Kg]$	Percentage [%]
Bottom	1484	49
Sides	851	28
Deck	726	23
TOT	3041	

(a) Matlab-based results

(b) Excel-based results

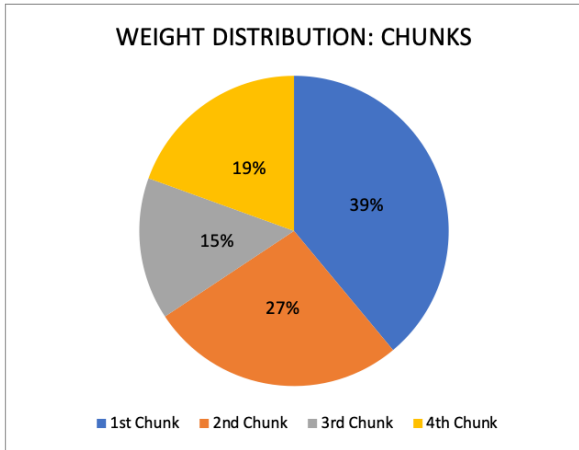Figure 6.7: Weight distribution (1 out of 3)

Table 6.33: Weight distributions (2 out of 3)
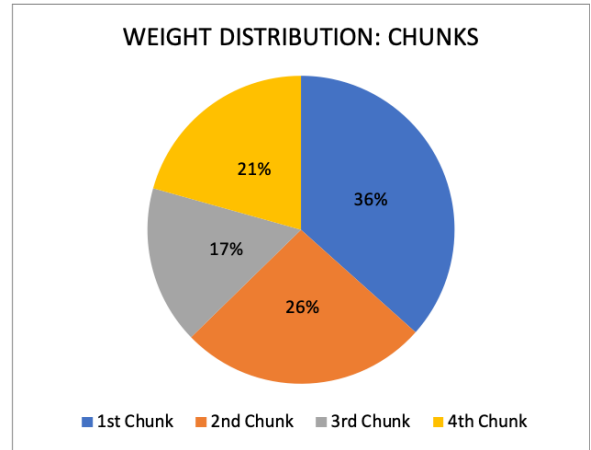
(a) Matlab-based results

Chunk	Weight $[Kg]$	Percentage [%]
1st Chunk	1033	39
2nd Chunk	710	27
3rd Chunk	393	15
4th Chunk	517	19
TOT	2653	

(b) Excel-based results

Chunk	Weight $[Kg]$	Percentage [%]
1st Chunk	1113	37
2nd Chunk	794	26
3rd Chunk	506	17
4th Chunk	628	21
TOT	3041	

(a) Matlab-based results   (b) Excel-based results
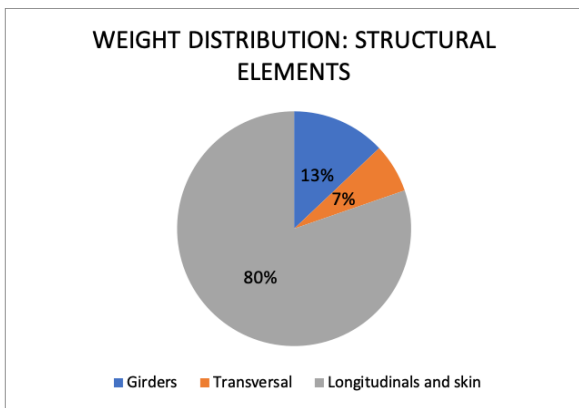
Figure 6.8: Weight distribution (2 out of 3)

Table 6.34: Weight distributions (3 out of 3)
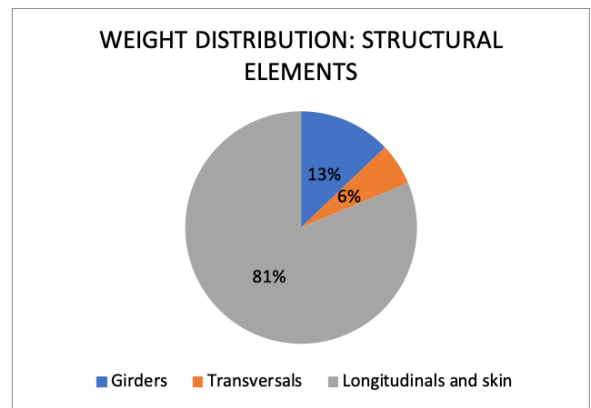
(a) Matlab-based results

Element	Weight $[Kg]$	Percentage $[\%]$
Girders	334	13
Transversal	179	7
Long. and skin	2130	80
TOT	2653	

(b) Excel-based results

Element	Weight $[Kg]$	Percentage $[\%]$
Girders	392	13
Transversal	178	6
Long. and skin	2471	81
TOT	3041	

(a) Matlab-based results   (b) Excel-based results

Figure 6.9: Weight distribution (3 out of 3)

The total weight of the structure has resulted to be approximately 2.6 tons, highlighting the power and effectiveness of the Matlab source with a 12% in weight saving respect to the Excel based optimizer. A part from this the general distribution of the weight along the different areas an elements of the structure results to be pretty similar within the two methods, highlighting how this aspect has never been taken into account. The weight saving moreover cannot be highlighted predominantly in a single chunk since these differ of almost 100 kg each. Different consideration can be made instead on the structural elements weight distribution. Tables 6.34 show in fact how the most save is focused around the girders, longitudinals and

skins, highlighting how effectively, being of a much lower resistance order, transversal elements are not really much optimisable.

Not very surprising is the trend described by the above 6.9 figure, which actually shows how the non-optimized skins are occupying the largest part of the total weight. On the same line, the weight distribution on the single chunk has resulted to be sufficiently balanced between bottom, sides and weather deck, despite having treated them completely separately.

6.7   Final thoughts and developments

Despite the large complications related to the coding of the Matlab source, the developed optimizer seems to be working effectively to save a large part of the original structure. It is not a new fact overall that Matlab offers the users functional tools created appositely to provide large computing power and easy to use functions in an all-in-one software. This being said, Excel results in most of the cases much more clear and useful for other engineering aspects such as charting and plotting, without even considering the ease of use with respect to the coding of a Matlab source like this one.

This cleared, from a personal stand point of view, it must be said that the developed software is very far from being perfect and ultimate. The beauty of use of Matlab overall encourages the user to enhance the coding, making always use of the provided functions which can reduce tremendously the computing timings. The method is also thought to be completely modular and adaptable to any case of study, for this reason this refining procedure shouldn't result to be very time consuming.

A large improvement of the whole experience of use may be to integrate the Excel file provided by the Professor M. Gaiotti directly into the Matlab workspace and to create apposite loops to let it use directly the optimized numerical regressions and find an optimal solution to the reinforced structural elements. This not only allows the user to commit to other matters, but speeds also up and improves even more the whole structural optimization procedure without even requiring the use of the Excel solver.

A minor improvement instead would be to integrate into the coding a nice working interface where the user can give Matlab all the necessary data required for the whole structural optimization, without intervening directly into the code itself each time and adjusting the parameters from case to case.

Overall the developed method has proofed large advantages of use with respect to the others, both in speed and results quality. For this reason the initial task to create an useful tool in order to understand the structural optimization mechanics has been reached and the present Master thesis can be considered concluded.

At least for now.

Appendix A

Ship General Plan